INVESTIGATION OF A SUBOPTIMAL CONTROLLER DESIGN FOR A NUCLEAR REACTOR SYSTEM

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THESIS

INVESTIGATION OF A SUBOPTIMAL CONTROLLER
DESIGN FOR A NUCLEAR REACTOR SYSTEM

by

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are chosen to minimize the derivations between the system and model responses using a search routine. The optimal feedback parameters computed for the second-order model is used for suboptimal control of the system. The model parameters are updated to reflect the system nonlinearities as well as changes in the system parameters; the corresponding control scheme is adaptive. It is shown that for the operating conditions considered, the adaptive controller need not be on-line.

Also, investigation of the effects of different weighting factors in the cost function, and the effect of various control rod configurations on the system response are presented.

Investigation of a Suboptimal Controller

Design for a Nuclear Reactor System

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TABLE OF SYMBOLS

Symbol	Definition
$n(t)$	neutron or power level, as a fraction of full power
t	time
$\delta k(t)$	reactivity
β	total delayed neutron fraction
ℓ	prompt neutron lifetime
λ_i	decay constant of i^{th} neutron precursor
$Ci(t)$	concentration of i^{th} neutron precursor
β_i	fraction of delay neutron due to i^{th} precursor
τ_t	time constant of temperature effect
α_t	temperature coefficient
δk_t	temperature reactivity
δk_c	reactivity due to absorber rod
δk_0	steady state value of δkc at $t=0$
T	time period of interest
V	motor velocity
G	apmplfier gain
τ_m	time constant of motor
e	power error

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I. INTRODUCTION

Since the advent of nuclear reactor technology, safety is generally a foremost consideration in the design of a nuclear reactor and its control system. To attain the purpose of safe operation with some improvement in performance, there is a possible need to apply optimal control theory. A considerable amount of research effort has been directed toward the realization of optimal control systems, especially for such big-scale projects as the operation of a nuclear power reactor. It is reasonable to assume that nuclear power stations of the future will be operated entirely under the control of digital computers, which is accepted as the most suitable device to perform such a task.

As mentioned by Sinha and Bereznai [Ref. 1], the over-simplified representation of the reactor dynamics, usually the one delayed neutron group model, is not satisfactory for control purposes, and their responses deviate considerably from that of the actual system. Therefore, the attempt is made to evaluate the response using a more complete representation of the reactor kinetics, and to include an adequate description of the controller mechanisms.

In this paper, a realistic system of a nuclear reactor, which is a ninth-order nonlinear and has time-varying parameters, is used. The measured quantity to be controlled

is the neutron or power level. Though any method that resolves the optimal control problem for this system is not known, it may be feasible to get a controller that will approximate the desired optimal response sufficiently close for practical purposes. A method that realizes such a suboptimal performance is described in this paper based on the work of Bereznai [Ref. 2]. It uses a second-order model to represent the reactor and the controller mechanism. The optimized parameters of the model are computed to approximate the response of the actual system using a search routine. Then, the feedback parameters required to use the suboptimal feedback controller for the nuclear reactor are easily computed. In this manner, the control system is adapted to compensate for the nonlinear system characteristics and for the changes in the system parameters.

II. SYSTEM REPRESENTATION

The mathematical model for the plant under consideration is developed. To retain sufficient accuracy, the reactor kinetics represented by the six delayed neutron group is used. The unity feedback is used to study the closed-loop response of the system.

A. REACTOR KINETICS

The kinetic equations of a chain-reacting pile have been derived in the literature many times [Ref. 3]. The six delayed neutron group, space independent, source free model is described by the following equations:

$$\frac{dn(t)}{dt} = \frac{\delta k(t) - \beta}{\ell} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) \qquad (2.1)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\ell} n(t) - \lambda_i C_i(t) \qquad (2.2)$$

where $i = 1, 2, \ldots, 6$.

All of these equations are linear in the so-called state variables, n and C_i ($i = 1, \ldots, 6$), and linear in the input, $\delta k(t)$, but the system is not jointly linear in state and control. Any feedback of n to formulate $\delta k(t)$ results in a nonlinear system.

B. TEMPERATURE REACTIVITY

In most reactors, heat-transfer dynamics are coupled to neutron kinetics by a temperature coefficient of reactivity which is generated by thermal expansion of the core and by change in various neutron cross sections. In other words, thermal expansion reduces density, and thereby the moderation ability and reactivity. Also, the neutron capture-to-fission ratio of the fuel is altered by temperature changes, and a smaller effect, due to changes in absorption and fission cross sections, increases leakage and decreases reactivity. Further, thermal-reactivity contribution results from so-called Doppler broadening associated with the resonance region of the neutron energy spectrum. The temperature coefficient α_t may usually be assumed to be a constant within the accuracy with which it can be predicted. Hence, the temperature reactivity may be expressed as:

$$\frac{d\delta k_t}{dt} = - \frac{\delta k_t}{\tau_t} + \frac{\alpha_t}{\tau_t} n(t) \quad . \tag{2.3}$$

The significance of the temperature coefficient of reactivity is obvious from Figure 2.1. Even if a linear heat-exchange model is utilized, the temperature-reactivity feedback causes the reactor dynamical model to be nonlinear. The effective temperature is assumed to be proportional to the operating power level.

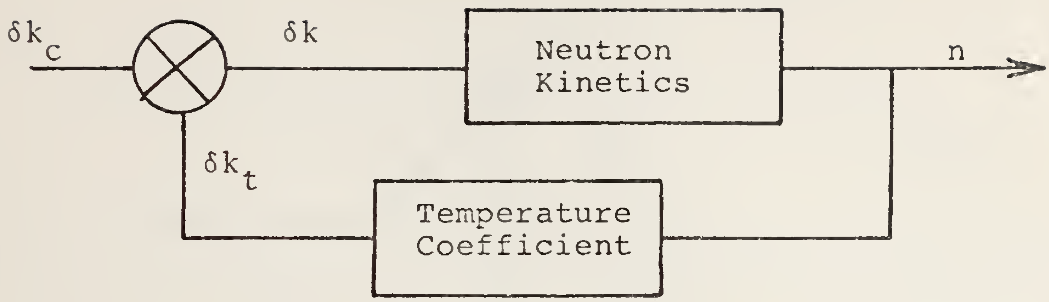


Figure (2.1)

Feedback Loop of Temperature-Reactivity

C. ABSORBER ROD

The reactivity term δk is the sum of the externally applied reactivity due to the absorber rod, and the change in the reactivity due to temperature variations. The mathematical representation of the reactivity due to the absorber rod is given in Reference 2:

$$\delta k_c = \delta k_0 + \int_0^T W(V)\, dt \tag{2.4}$$

$$\text{where } W(V) = \begin{cases} 0.02\, V(t) & |V| \leq 15 \\ 0.3 & V > 15 \\ -0.3 & V < -15 \end{cases} \tag{2.5}$$

that is:

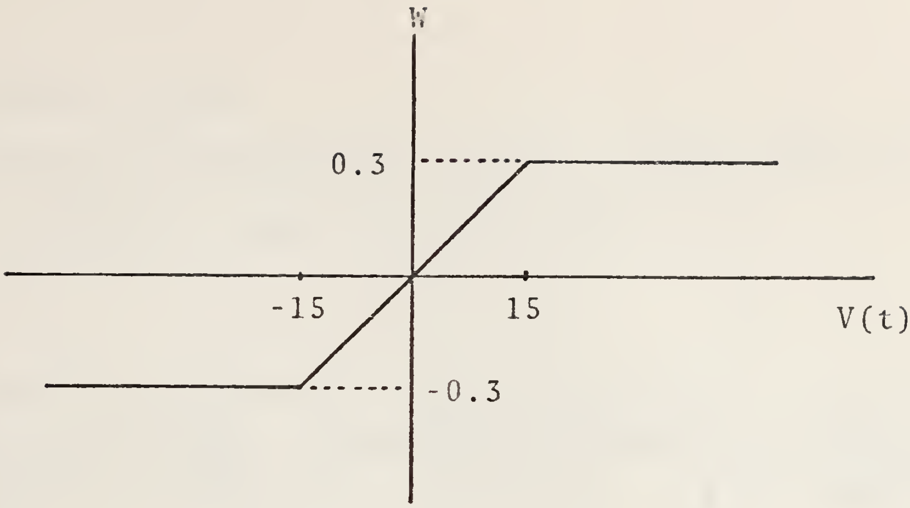


Figure (2.2)

Absorber Rod Characteristic Curve

W(V) has the units mk/sec, and represents the reactivity change due to the motion of the absorber rod. This formulation assumes that the control rod is near the center of its movement, where the reactivity changes linearly with distance from the center of the core.

D. DRIVE MOTOR

The time response of the drive motor is given by:

$$\frac{dV}{dt} = \frac{-V}{\tau_m} + \frac{G}{\tau_m} e \qquad (2.6)$$

where G is gain, V is motor velocity, e is power error and τ_m is time constant of motor.

E. OVERALL SYSTEM

The block diagram of the nuclear reactor model considered in this thesis is represented in Figure (2.3).

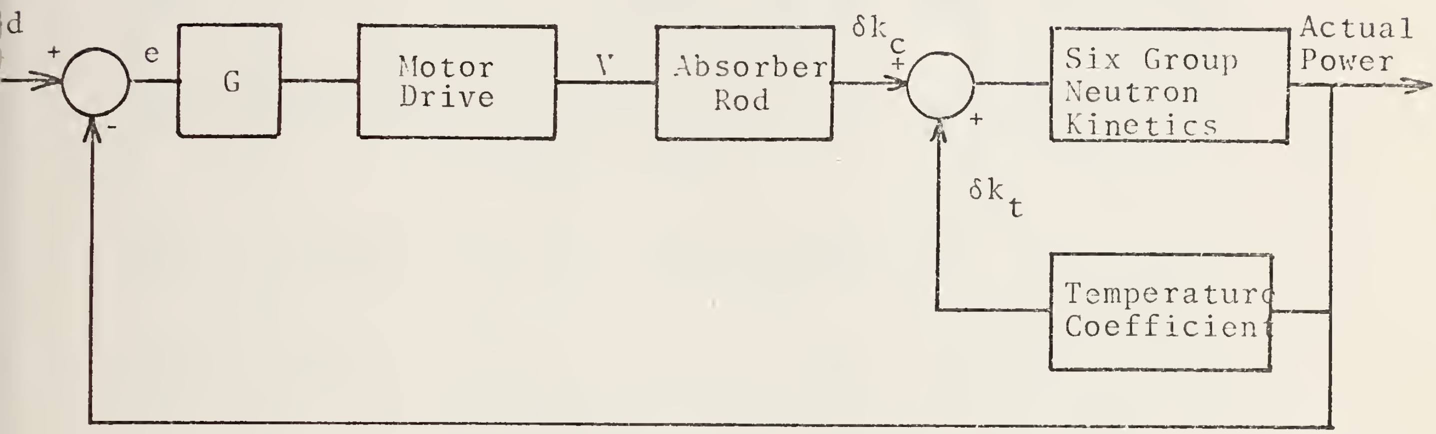


Figure (2.3)

Block Diagram of Nuclear Reactor and Reactivity Mechanism

In order to study the closed-loop response of the system, unity feedback is used. The error signal (e) that expresses the deviation between the demanded and actual power levels is amplified, and the output is applied to the absorber rod drive motor. The rate of reactivity insertion (δk_c/sec) is proportional to the error signal until the motor reaches its maximum speed. The actual reactivity δk, which is a measure of the criticality or multiplication factor of the reactor, is the difference between the reactivity worth of the absorber rod and the effect of the temperature change.

In state variable form the system equations that represent the reactor and the absorber rod mechanism in an open loop configuration are written in the following:

$$
\begin{pmatrix} \frac{dn}{dt} \\ \frac{dC_1}{dt} \\ \frac{dC_2}{dt} \\ \frac{dC_3}{dt} \\ \frac{dC_4}{dt} \\ \frac{dC_5}{dt} \\ \frac{dC_6}{dt} \\ \frac{d\delta k_t}{dt} \\ \frac{dV}{dt} \end{pmatrix}
=
\begin{pmatrix}
-\frac{\beta}{\ell} & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 & \frac{n}{\ell} & \frac{ntW}{\ell V} \\
\frac{\beta_1}{\ell} & -\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\beta_2}{\ell} & 0 & -\lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\beta_3}{\ell} & 0 & 0 & -\lambda_3 & 0 & 0 & 0 & 0 & 0 \\
\frac{\beta_4}{\ell} & 0 & 0 & 0 & -\lambda_4 & 0 & 0 & 0 & 0 \\
\frac{\beta_5}{\ell} & 0 & 0 & 0 & 0 & -\lambda_5 & 0 & 0 & 0 \\
\frac{\beta_6}{\ell} & 0 & 0 & 0 & 0 & 0 & -\lambda_6 & 0 & 0 \\
\frac{\alpha_t}{\tau_t} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{\tau_t} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{\tau_m}
\end{pmatrix}
\begin{pmatrix} n \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ \delta k_t \\ V \end{pmatrix}
+
\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{G}{\tau_m} \end{pmatrix} e
\tag{2.7}
$$

Using matrix notation

$$\dot{\underset{\sim}{x}} = \underset{\sim}{A}\underset{\sim}{x} + \underset{\sim}{B}e \tag{2.8}$$

and the system output is given by

$$y = \underset{\sim}{C}^T \underset{\sim}{x} \tag{2.9}$$

where $\underset{\sim}{C}^T = [1,0\ldots0]$.

The numerical values of the parameters are given in Appendix A. Hence, the parameters of reactor kinetics equations are typical values, and the parameters of the absorber rod mechanism are given in Reference 4.

III. DYNAMICAL BEHAVIOR OF SYSTEM

The equations of the system described in Section II were solved on an IBM 360, Model 67 digital computer (W. R. Church Computer Center at the Naval Postgraduate School) using the method of digital simulation language (DSL), which has been suitably modified to take into account the nonlinear nature of the differential equations.

A. CLOSED-LOOP RESPONSE OF NUCLEAR REACTOR

The closed-loop responses of the nuclear reactor to step-change in demanded power level, for various initial power level are shown in Figure (3.1) to Figure (3.8). The reference points such as the time to reach first full power (100% FP), the amplitude and the time of maximum overshoot are tabulated in Table (3.1). The nonlinear characteristic of the system is clearly shown in the figures.

B. EFFECT OF CONTROL-ROD TO THE RESPONSE OF NUCLEAR REACTOR

Various control-rod configurations are tried to examine the effect to the system response. These are shown in Table (3.2). Hence, the number given to the control-rod shown in Table (3.2) are corresponding to the number of Figures (3.9) and (3.10).

The configurations of 1, 2 and 3 have the same W_{max} value and different slopes for the linear region. These system responses are shown in Figure (3.9). In this case,

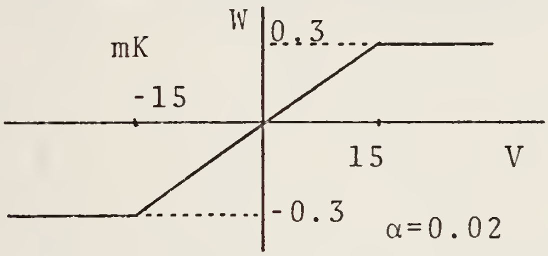
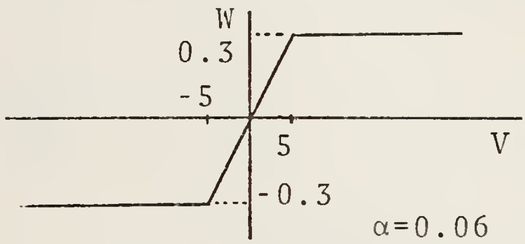
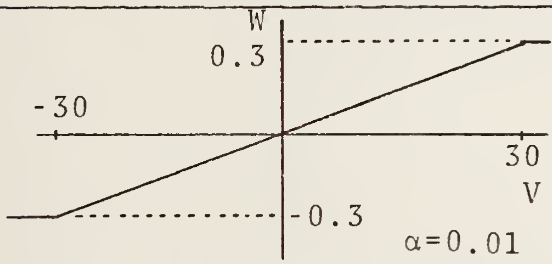
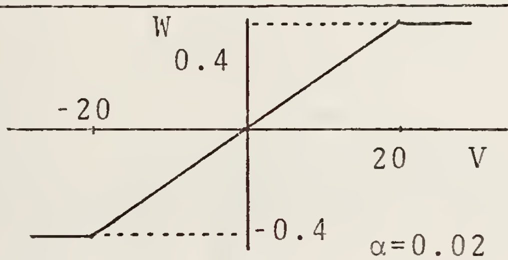
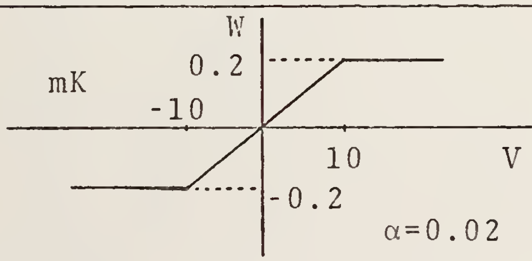
Initial Power Level	Time to Reach 100% Power Level	Maximum Overshoot	
		Amplitude	Time
95%	1.16 sec.	101.05%	1.59 sec
90%	2.03	101.44%	2.48
80%	3.59	102.03%	4.05
60%	6.53	103.07%	7.05
50%	7.99	103.81%	8.56
30%	10.97	106.36%	11.76
20%	12.64	109.33%	13.66

Table (3.1)

Comparison of Some Reference Points to Various Initial Power Levels

Table (3.2)

Control Rod Configurations

No.	Configuration	Equation
1		$W = 0.02V \quad \lvert V\rvert \leq 15$ $= 0.3 \quad V > 15$ $= -0.3 \quad V < -15$
2		$W = 0.06V \quad \lvert V\rvert \leq 5$ $= 0.3 \quad V > 5$ $= -0.3 \quad V < -5$
3		$W = 0.01V \quad \lvert V\rvert \leq 30$ $= 0.3 \quad V > 30$ $= -0.3 \quad V < -30$
4		$W = 0.02V \quad \lvert V\rvert \leq 20$ $= 0.4 \quad V > 20$ $= -0.4 \quad V < -20$
5		$W = 0.02V \quad \lvert V\rvert \leq 10$ $= 0.2 \quad V > 10$ $= -0.2 \quad V < -10$

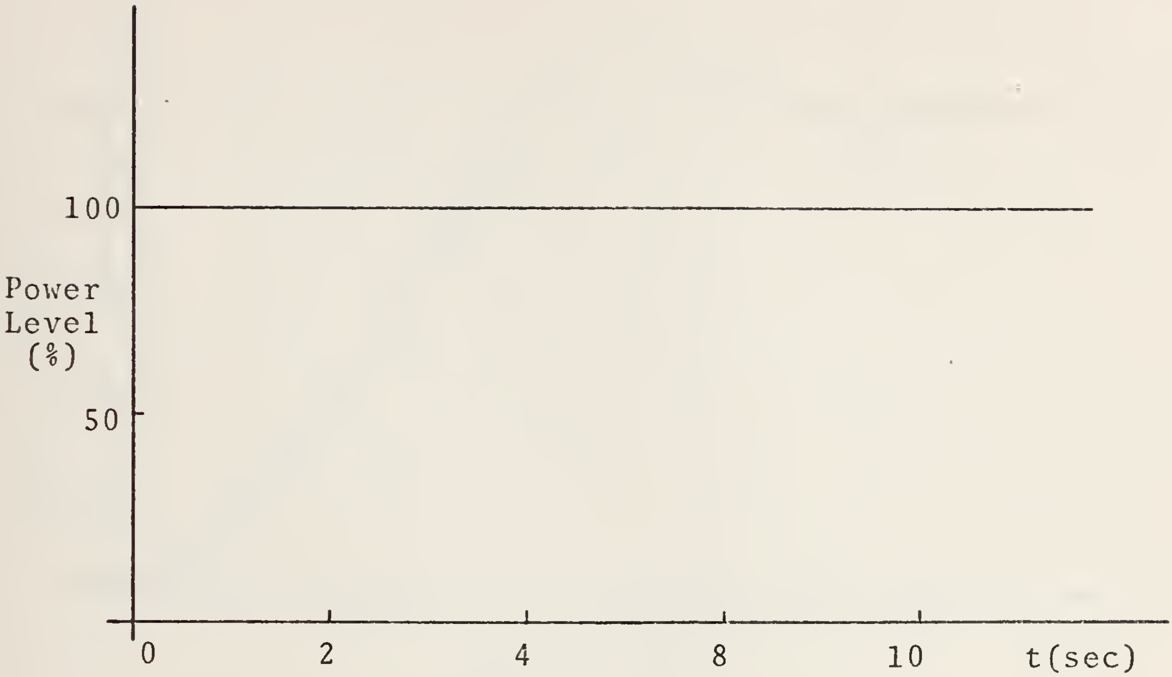


Figure (3.1)
Steady State Response

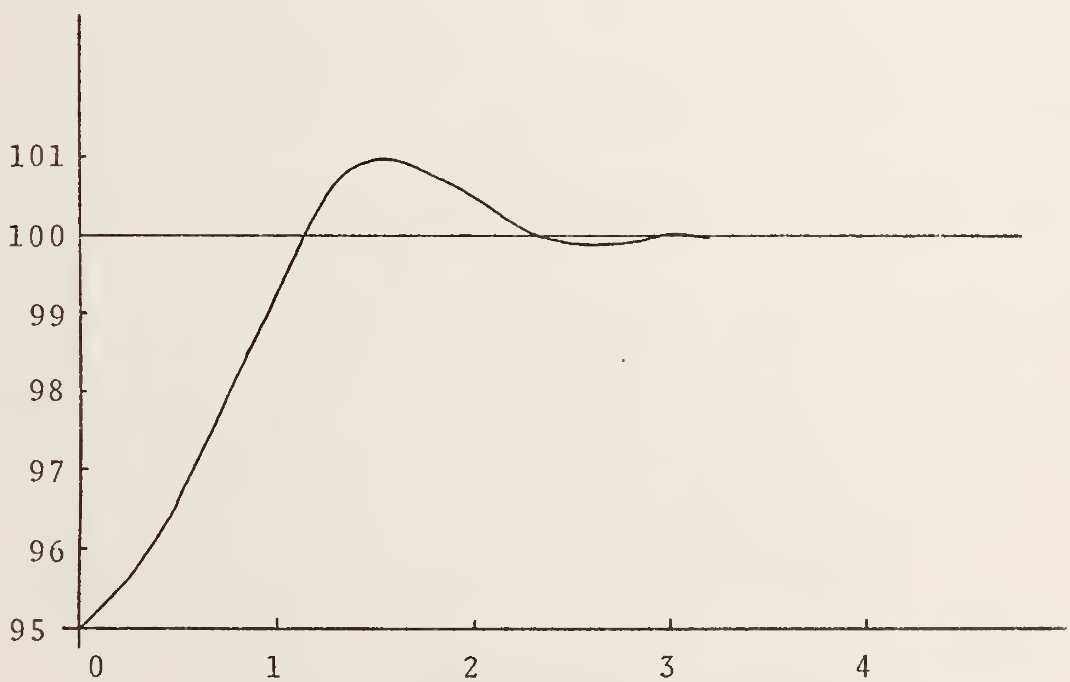


Figure (3.2)
Step Response of Initial Power Level 95%

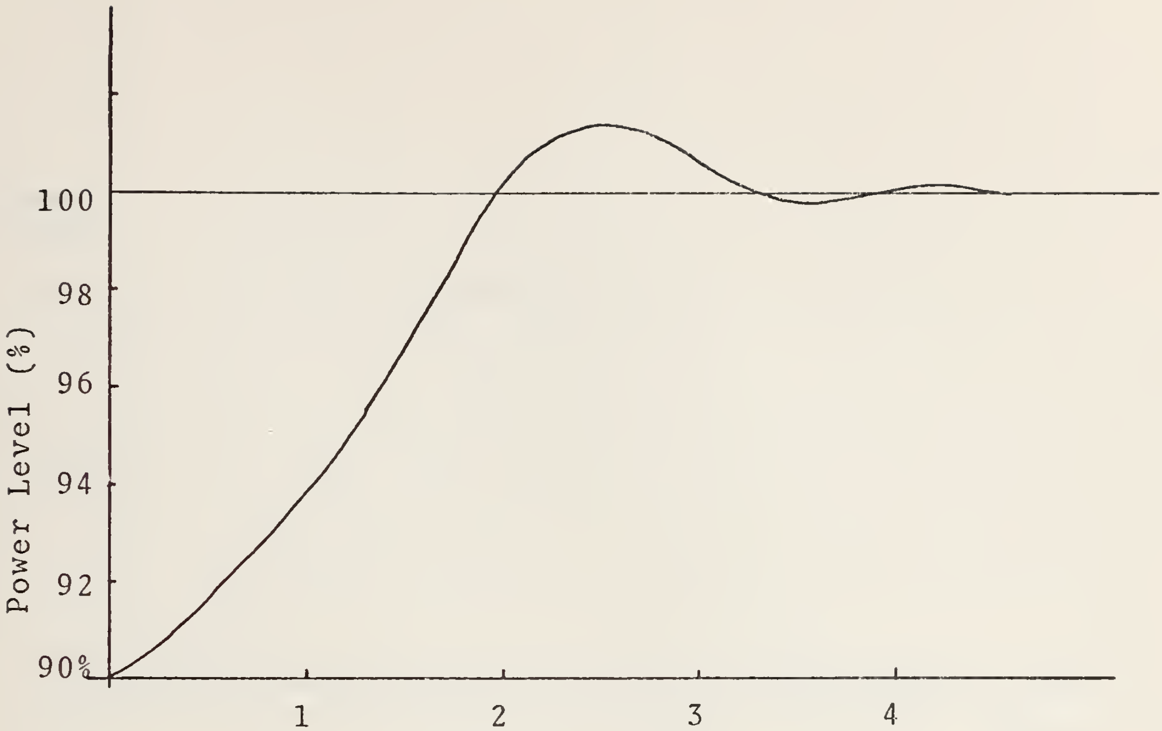


Figure (3.3)

Step Response of Initial Power Level 90%

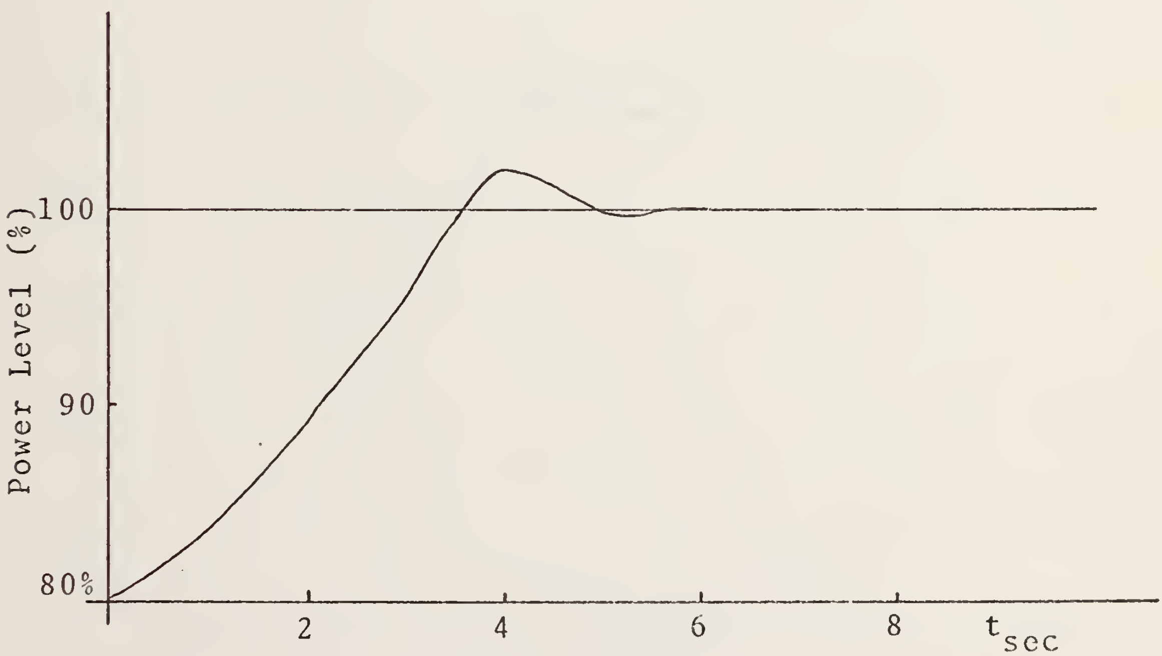


Figure (3.4)

Step Response of Initial Power Level 80%

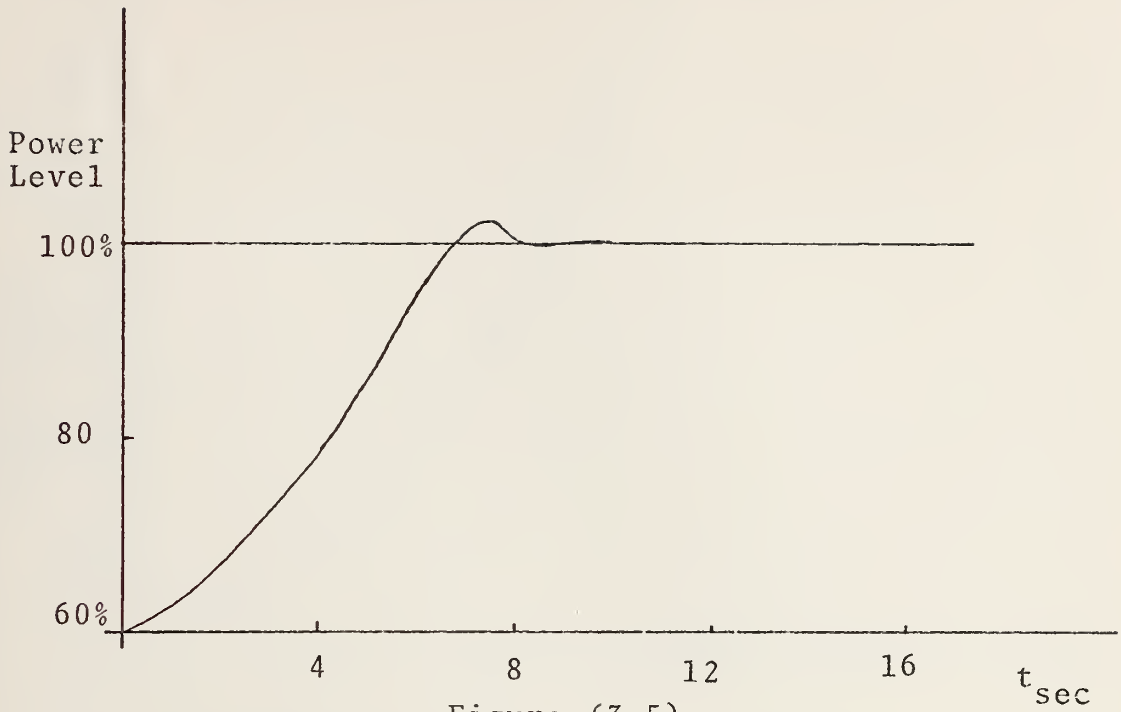


Figure (3.5)
Step Response of Initial Power Level 60%

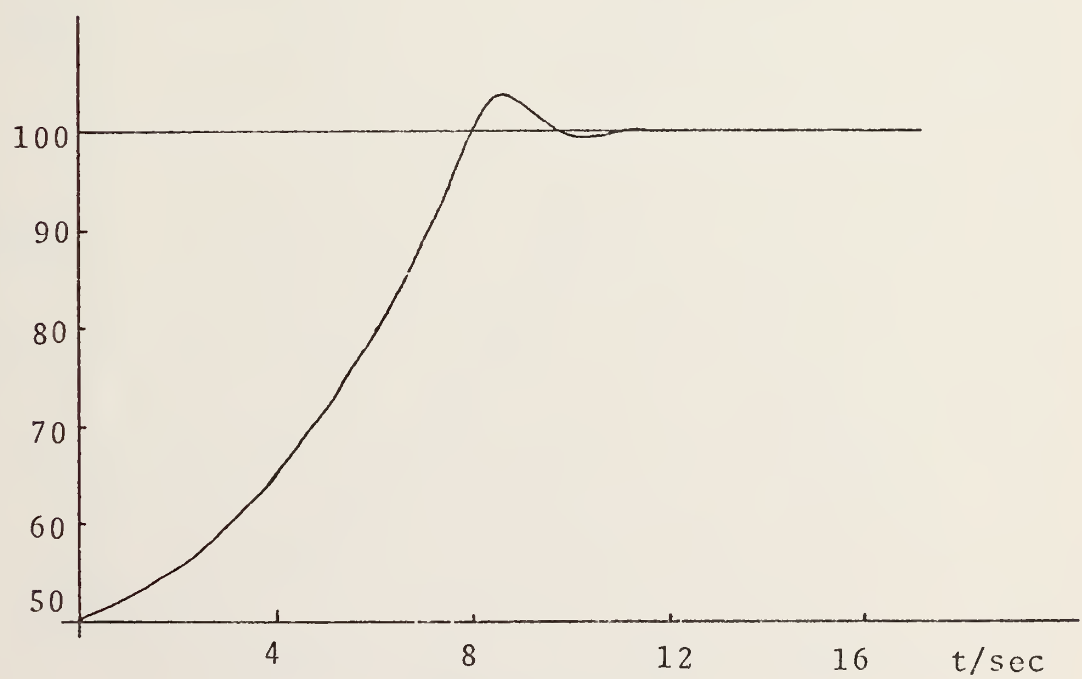


Figure (3.6)
Step Response of Initial Power Level 50%

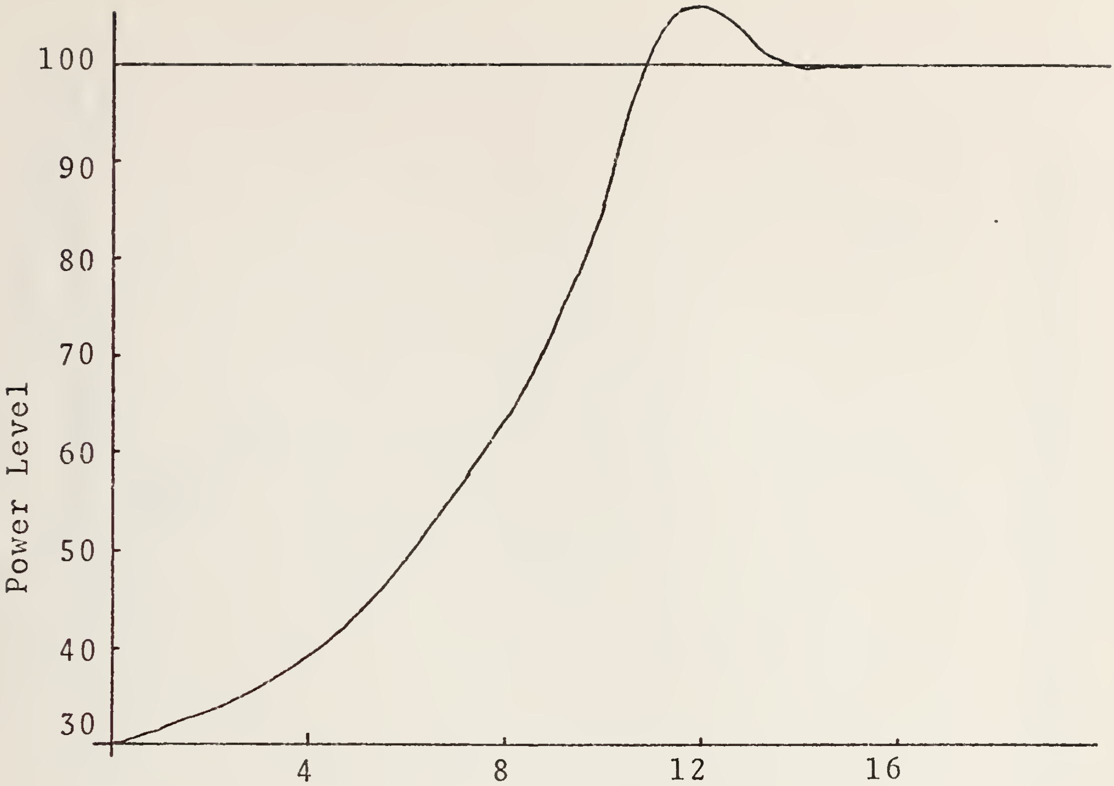


Figure (3.7)
Step Response of Initial
Power Level 30%

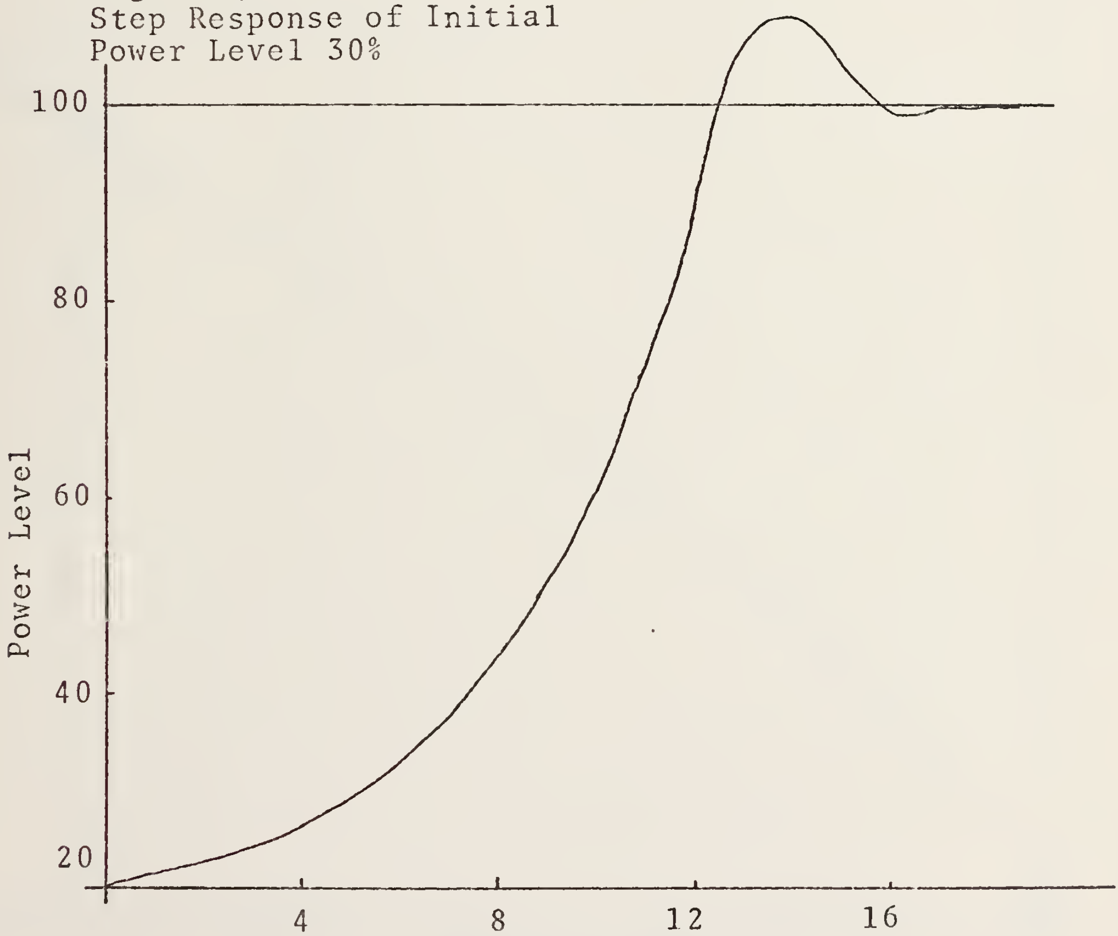


Figure (3.8)
Step Response of Initial Power Level 20%

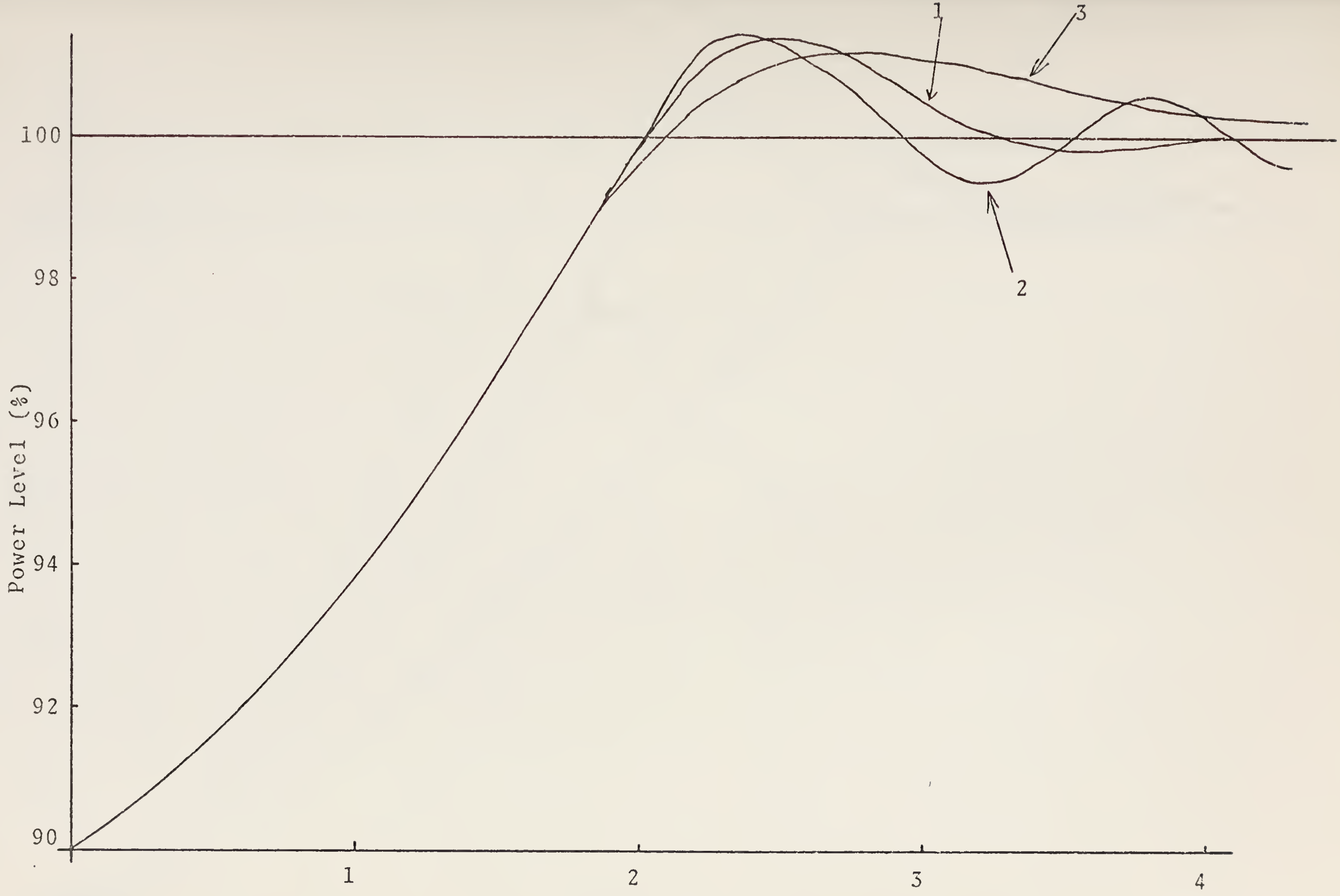


Figure (3.9)

Step Response of Reactor System Using Control Rods Designated 1, 2 and 3

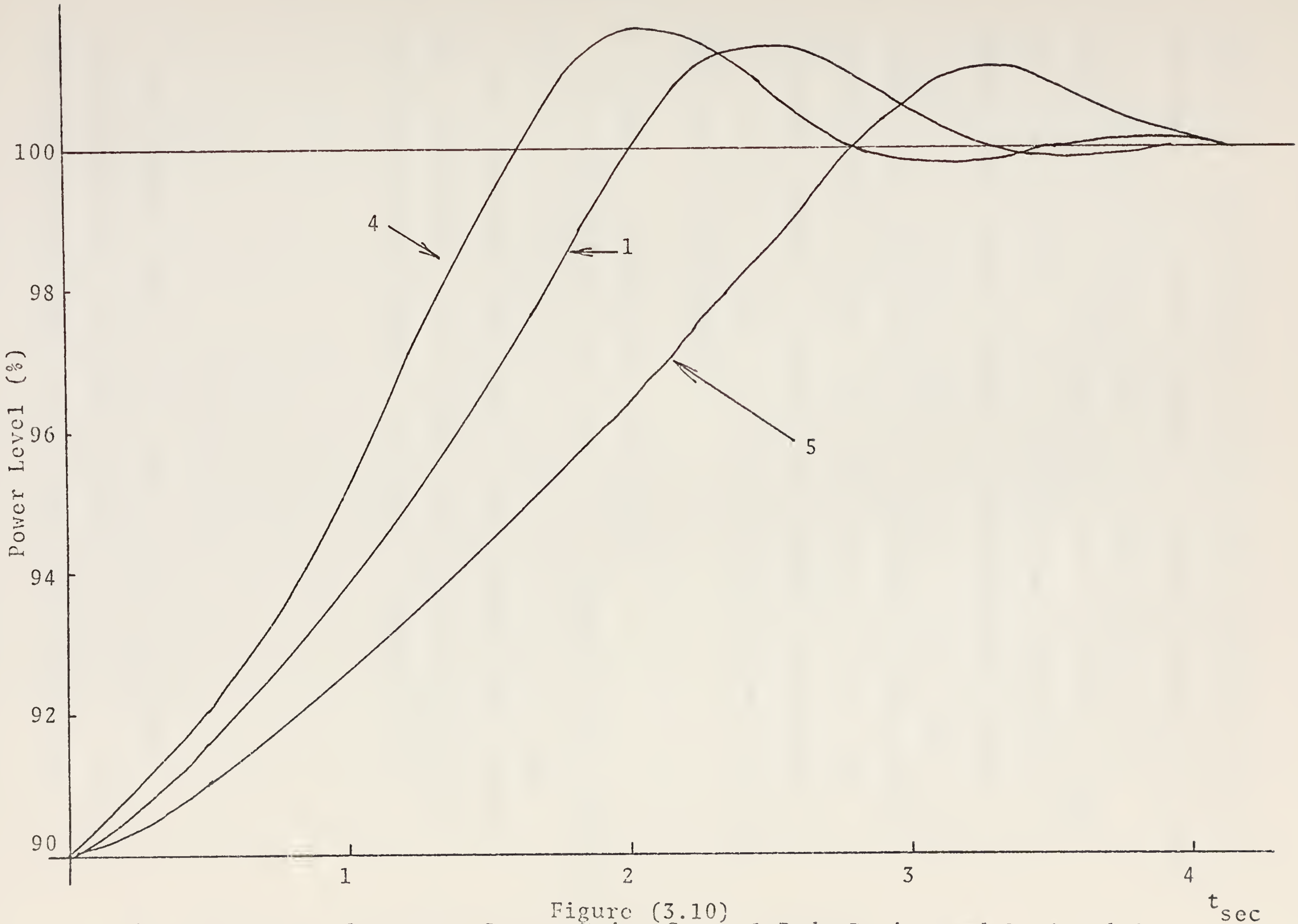


Figure (3.10)
Step Response of Reactor System Using Control Rods Designated 1, 4 and 5

the early system response for all three configurations are identical. On the other hand, the configurations of 1, 4 and 5 have different W_{max} values. As understandable from Figure (3.10), the different W_{max} give much effect to the early time response. So, the control-rod characteristic becomes the important factor in determining the system response for the early values of time. In this paper, the control-rod characteristic of 1, which has an application in a particular reactor study [Ref. 2], is used in the following sections. In addition, a faster rod response characteristic is used in Section D where maximum reactivity insertion is investigated.

C. EFFECT OF TEMPERATURE COEFFICIENT

The temperature coefficient α_t is assumed to be a constant, but in a practical reactor it may vary over some range.

To investigate the effect of some range of the temperature coefficient, the following temperature coefficients are used and the responses are shown in Figure (3.11).

$$\alpha_t = -8 \times 10^{-3}$$
$$\alpha_t = -3.5 \times 10^{-3}$$
$$\alpha_t = 1 \times 10^{-3}$$
$$\alpha_t = 10 \times 10^{-3}$$

As the figures indicate, for values of the temperature coefficient about the nominal value of -3.5×10^{-3}, these temperature coefficients do not have much influence to

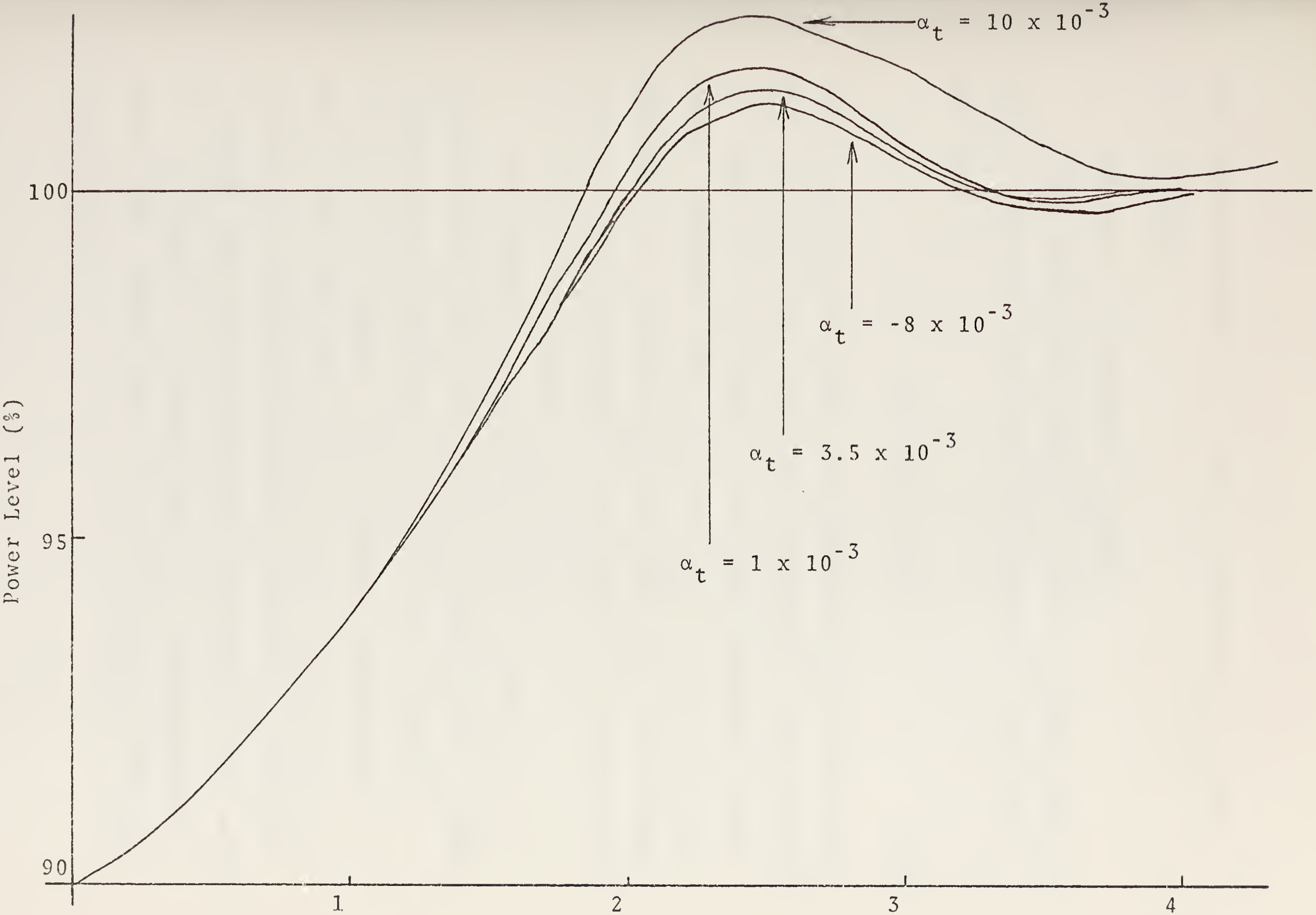


Figure (3.11)
Effect of Temperature Coefficient

the system response. It works to increase or decrease the overshoot.

D. CONSTRAINT OF REACTIVITY INSERTION

Total reactivity δk which is the sum of the reactivity due to the control rod, and the temperature reactivity is expressed as:

$$\delta k = \delta k_c + \delta k_t \tag{3.1}$$

In the previous sections, the magnitude of the reactivity was not constrained. As shown in Section C, the magnitude of temperature reactivity is not very influential in the system response. The interested point is the constraint applied to the control-rod reactivity rate. The movement of the control-rod should be restricted to a maximum value from physical viewpoints.

Now, the graph of the control-rod reactivity of the initial power level 80% is shown in Figure (3.12). Hence, as described in Section B in Chapter III, the faster control-rod is chosen. Three different constraints of control-rod reactivity are considered. These three constraints are interesting points, because the first works to reduce both the first and the second overshoots, and the third constraint is entirely below the actual reactivity except the starting area. These situations are shown in Figure (3.12).

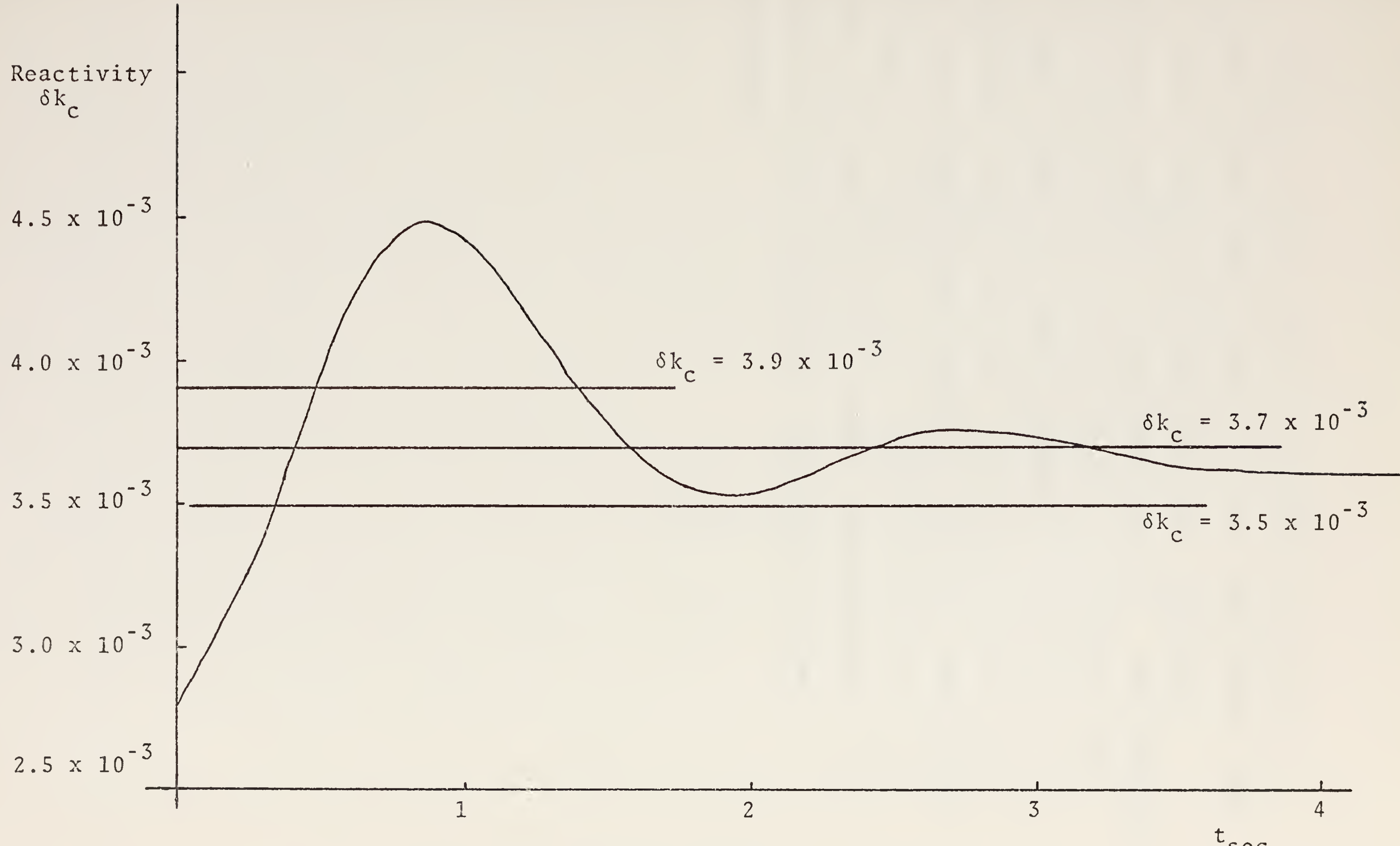


Figure (3.12)
Reactivity of Control Rod for 80 to 100% Power Change

The Figure (3.13) shows the relation of the system response and the control-rod reactivity. The obvious fact is that the control-rod reactivity gives much effect, especially stability, to the system response. The influence of the constraints of the control-rod reactivity is shown in Figure (3.14). As shown in Figure (3.14), the constraint of $\delta k_c = 3.9 \times 10^{-3}$ is the most effective one, and the desirable system response of fast rise and minimum overshoot is obtained. So the suitable constraint on the reactivity insertion of the control rod contributes to the stability of the system while improving the system performance.

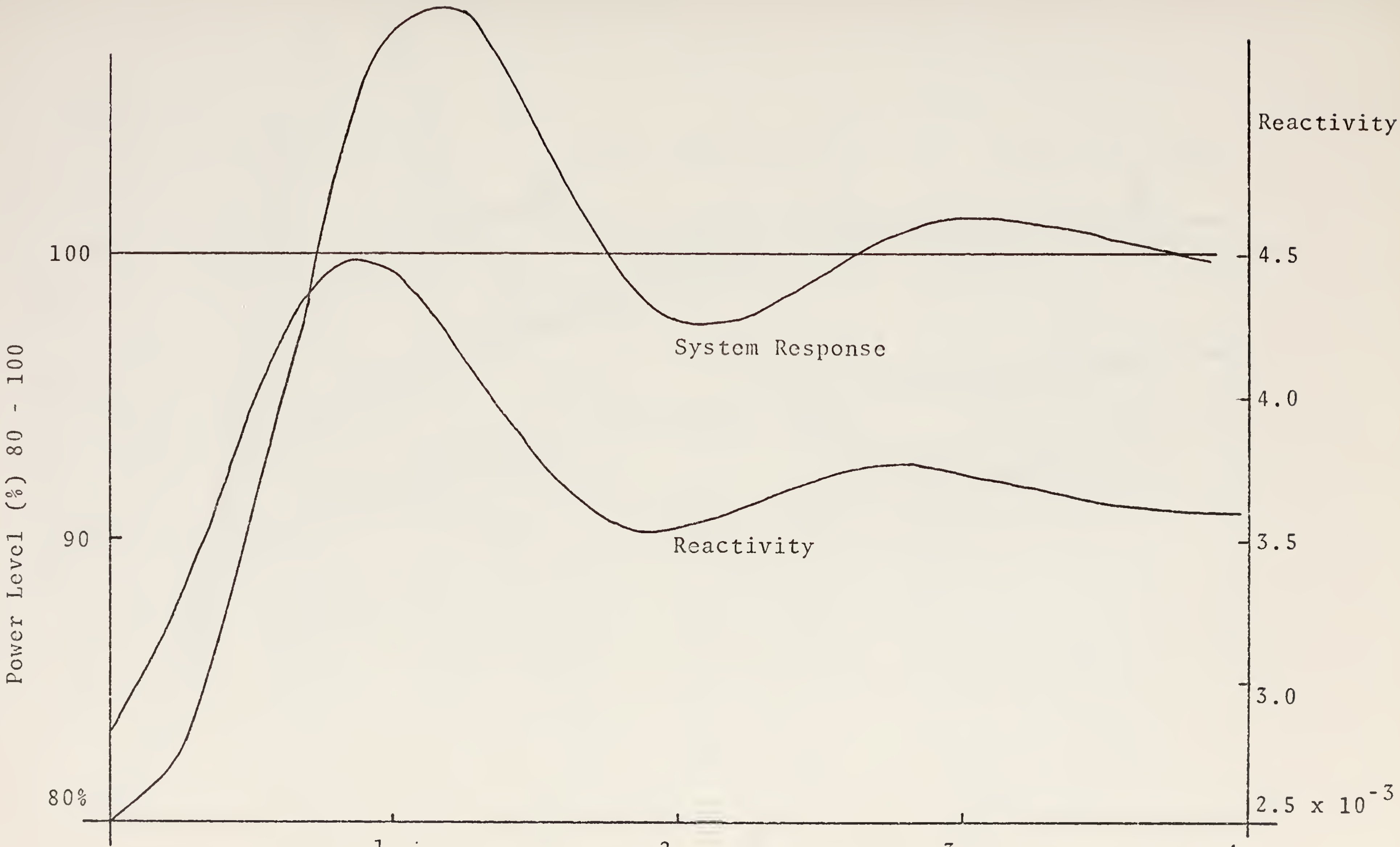


Figure (3.13)
Relation of System Response and Reactivity

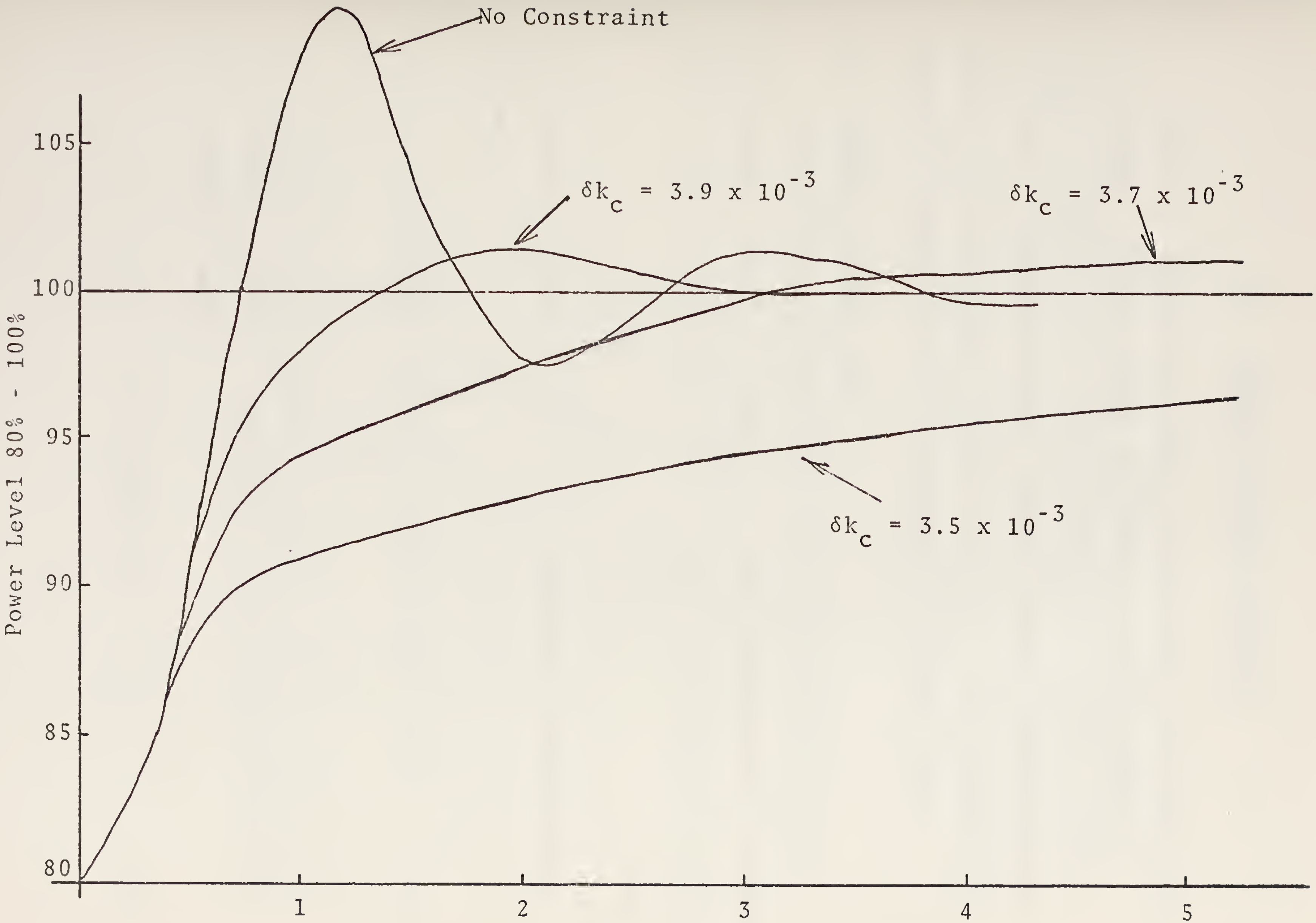


Figure (3.14)
Influence of Constraints

IV. OPTIMAL CONTROL PROBLEM

There is a possible need to investigate the application of optimal control theory to nuclear reactor system in order to improve the system performance. In this thesis the realization of optimal control systems will be investigated for step demand changes in power. The procedure is, however, applied for any deterministic demand input.

A. OPTIMAL CONTROL LAW

The optimal control theory has been derived in many literatures [Refs. 5, 6 and 7]. The optimal control problem is to find a control $\underset{\sim}{\mu}^* \varepsilon U$ which causes the nonlinear system

$$\dot{\underset{\sim}{x}}(t) = f(\underset{\sim}{x}(t), \underset{\sim}{\mu}(t), t) \qquad (4.1)$$

to follow a trajectory $\dot{\underset{\sim}{x}}^* \varepsilon X$ that minimizes the performance measure

$$J = h(\underset{\sim}{x}(t_f), t_f) + \int_{t_0}^{t_f} g(\underset{\sim}{x}(t), \underset{\sim}{\mu}(t), t)\, dt \qquad (4.2)$$

For most cases of practical viewpoint, such as the control of a nuclear reactor, the optimal control must be realized in a feedback manner,

$$\underset{\sim}{\mu} = f(\underset{\sim}{x}(t), t) \qquad (4.3)$$

While the mathematical solution of this problem is established, it involves the solution of a nonlinear partial differential equation.

Optimal-controller synthesis is complicated by the nonlinear model which is utilized to approximate reactor dynamics. For linear systems, however, it is possible to obtain analytical solutions to such problems. Further, it is shown in this section that the synthesis of the control which minimizes an integrated quadratic in state and control only requires a linear feedback of state variables.

Assume for convenience that the dynamical behavior of the system is approximated by the linear model

$$\dot{\underset{\sim}{x}}(t) = \underset{\sim}{A}(t)\ \underset{\sim}{x}(t) + \underset{\sim}{B}(t)\ \underset{\sim}{\mu}(t) \tag{4.4}$$

$$\text{where } \underset{\sim}{x}(t_0) = \underset{\sim}{x}_0$$

The problem is to find the admissible $\underset{\sim}{\mu}(t)$, or, preferably, $\underset{\sim}{\mu}(\underset{\sim}{x})$, which controls equation (4.4) with respect to some reference $\underset{\sim}{x}^*(t)$ so as to minimize

$$J = \int_{t_o}^{t_f} f_o(\underset{\sim}{x}(t),\ \underset{\sim}{\mu}(t),\ t)dt \tag{4.5}$$

where

$$f_o(\underset{\sim}{x}(t),\ \underset{\sim}{\mu}(t),\ t) = \frac{1}{2}\ [(\underset{\sim}{x}^*-\underset{\sim}{x})^T \cdot \underset{\sim}{Q}(t)(\underset{\sim}{x}^*-\underset{\sim}{x})$$

$$+ \underset{\sim}{\mu}^* \cdot \underset{\sim}{R}(t)\ \underset{\sim}{\mu}] \tag{4.6}$$

and R(t) is a real symmetric positive-definite matrix and Q(t) is a positive-semidefinite matrix.

Obviously, this problem may be solved by the maximum principle (or Hamilton's equations), or by dynamic programming (or the Hamilton-Jacobi-Bellman equations). To use the Hamilton-Jacobi-Bellman equation, the Hamiltonian is formed:

$$H(\underset{\sim}{x}(t),\ \underset{\sim}{\mu}(t),\ \frac{\partial J}{\partial \underset{\sim}{x}},\ t)$$

$$= \frac{1}{2}\,[\underset{\sim}{x}^T(t)\ \underset{\sim}{Q}(t)\ \underset{\sim}{x}(t) + \underset{\sim}{\mu}^T(t)\ \underset{\sim}{R}(t)\ \underset{\sim}{\mu}(t)]$$

$$+ \frac{\partial J^T}{\partial \underset{\sim}{x}} \cdot [A(t)\ \underset{\sim}{x}(t) + \underset{\sim}{B}(t)\ \underset{\sim}{\mu}(f)] \tag{4.7}$$

A necessary condition for $\underset{\sim}{\mu}(t)$ to minimize H is that $\frac{\partial H}{\partial \underset{\sim}{\mu}} = 0$; thus

$$\frac{\partial H}{\partial \underset{\sim}{\mu}}\ (\underset{\sim}{x}(t),\ \underset{\sim}{\mu}(t),\ \frac{\partial J}{\partial \underset{\sim}{x}},\ t)$$

$$= \underset{\sim}{R}(t)\ \underset{\sim}{\mu}(t) + \underset{\sim}{B}^T(t)\ \frac{\partial J}{\partial \underset{\sim}{x}} = 0 \tag{4.8}$$

Since the matrix

$$\frac{\partial^2 H}{\partial \underset{\sim}{\mu}^2} = \underset{\sim}{R}(t) \tag{4.9}$$

is positive definite and H is a quadratic form in $\underset{\sim}{\mu}$. the control that satisfies equation (4.8) does minimize H.

Solving equation (4.8) for $\underset{\sim}{\mu}^*(t)$ gives

$$\mu^*(t) = -\underset{\sim}{R}^{-1}(t)\ B^T(t)\ \frac{\partial J}{\partial \underset{\sim}{x}} \tag{4.10}$$

which when substituted in equation (4.7) yields

$$H(\underset{\sim}{x}(t),\ \mu^*(t),\ \frac{\partial J}{\partial \underset{\sim}{x}}\ ,\ t) = \frac{1}{2}\ \underset{\sim}{x}^T\ \underset{\sim}{Q}\ \underset{\sim}{x} - \frac{1}{2}\ \frac{\partial J^T}{\partial \underset{\sim}{x}}\ \underset{\sim}{B}\ \underset{\sim}{R}^{-1}\ B^T\ \frac{\partial J}{\partial \underset{\sim}{x}}$$

$$+ \frac{\partial J^J}{\partial \underset{\sim}{x}}\ \underset{\sim}{A}\ \underset{\sim}{x} \tag{4.11}$$

the Hamilton-Jacobi-Bellman equation is

$$\frac{\partial J(\underset{\sim}{x},t)}{\partial t} + \frac{1}{2}\ \underset{\sim}{x}^T\ \underset{\sim}{Q}\ \underset{\sim}{x} - \frac{1}{2}\ \frac{\partial J(\underset{\sim}{x},t)^T}{\partial x}\ \underset{\sim}{B}\ \underset{\sim}{R}^{-1}\ \underset{\sim}{B}^T\ \frac{\partial J(\underset{\sim}{x},t)}{\partial x}$$

$$+ \frac{\partial J(\underset{\sim}{x},t)^T}{\partial x}\ \underset{\sim}{A}\ \underset{\sim}{X} \tag{4.12}$$

From equation (4.5) the boundary condition is

$$J\ (\underset{\sim}{x}(t_f)\ ,\ t_f) = 0 \tag{4.13}$$

Since the minimum cost for the discrete linear regulator problem is a quadratic function of the state, it seems reasonable to guess as a solution of the form [Ref. 5]

$$J\ (\tilde{x}(t),\ t) = \frac{1}{2}\ \underset{\sim}{x}^T\ (t)\ \underset{\sim}{K}(t)\ \underset{\sim}{x}(t) \tag{4.14}$$

where $\underset{\sim}{K}(t)$ is a real symmetric positive-definite matrix that is to be determined. Substituting this assumed solution in equation (4.12) yields the result

$$\frac{1}{2} \underset{\sim}{x}^T \dot{\underset{\sim}{K}} \underset{\sim}{x} + \frac{1}{2} \underset{\sim}{x}^T \underset{\sim}{Q} \underset{\sim}{x} - \frac{1}{2} \underset{\sim}{x}^T \underset{\sim}{K} \underset{\sim}{B} \underset{\sim}{R}^{-1} \underset{\sim}{B}^T \underset{\sim}{K} \underset{\sim}{x} + \underset{\sim}{x}^T \underset{\sim}{K} \underset{\sim}{A} \underset{\sim}{x} = 0 \quad (4.15)$$

where generally all these matrices and vectors are functions of time. Equation (4.15) must be valid for all $\underset{\sim}{x}(t)$. Using the matrix property, this equation becomes:

$$\dot{\underset{\sim}{K}}(t) + \underset{\sim}{K}(t) \underset{\sim}{A}(t) + \underset{\sim}{A}^T(t) \underset{\sim}{K}(t)$$

$$-\underset{\sim}{K}(t) \underset{\sim}{B}(t) \underset{\sim}{R}^{-1}(t) \underset{\sim}{B}^T(t) \underset{\sim}{K}(t) + \underset{\sim}{Q}(t) = 0 \quad (4.16)$$

and the boundary condition is

$$\underset{\sim}{K}(t_f) = 0 \quad (4.17)$$

Once $\underset{\sim}{K}(t)$ has been determined, the optimal control law is given by

$$\underset{\sim}{\mu}^*(t) = -\underset{\sim}{R}^{-1}(t) \underset{\sim}{B}^T(t) \underset{\sim}{K}(t) \underset{\sim}{x}(t) \quad (4.18)$$

If the system is completely controllable, it is sometimes desirable to let $t_f \to \infty$, so that the optimal control accurately maintains the desired terminal state once it is reached. Further, Kalman [Ref. 8] has shown that if

$$J = \frac{1}{2} \int_0^\infty (\underset{\sim}{x}^T(t) \underset{\sim}{Q} \underset{\sim}{x}(t) + \underset{\sim}{\mu}^T(t) \underset{\sim}{R} \underset{\sim}{\mu}(t))dt \quad (4.19)$$

where $\underset{\sim}{Q}$ and $\underset{\sim}{R}$ are positive-definite, constant, symmetrical matrices and the system [equation (4.4)] is time invariant

($\underset{\sim}{A}$ and $\underset{\sim}{B}$ are constant matrices), then

$$\lim_{t\to\infty} \frac{d\underset{\sim}{K}(t)}{dt} = 0 \tag{4.20}$$

In this case, equation (4.16) becomes

$$\underset{\sim}{K}\,\underset{\sim}{A} + \underset{\sim}{A}^T \underset{\sim}{K} - \underset{\sim}{K}\,\underset{\sim}{B}\,\underset{\sim}{R}^{-1} \underset{\sim}{B}^T \underset{\sim}{K} + \underset{\sim}{Q} = 0 \tag{4.21}$$

For this problem, the optimal control is

$$\underset{\sim}{\mu}^*(\underset{\sim}{x}) = -\underset{\sim}{R}^{-1} \underset{\sim}{B}^T \underset{\sim}{K}\,\underset{\sim}{x} = -\underset{\sim}{k}^T \underset{\sim}{x} \tag{4.22}$$

and the synthesis requires a constant linear feedback of all the state variables as shown in Figure (4.1).

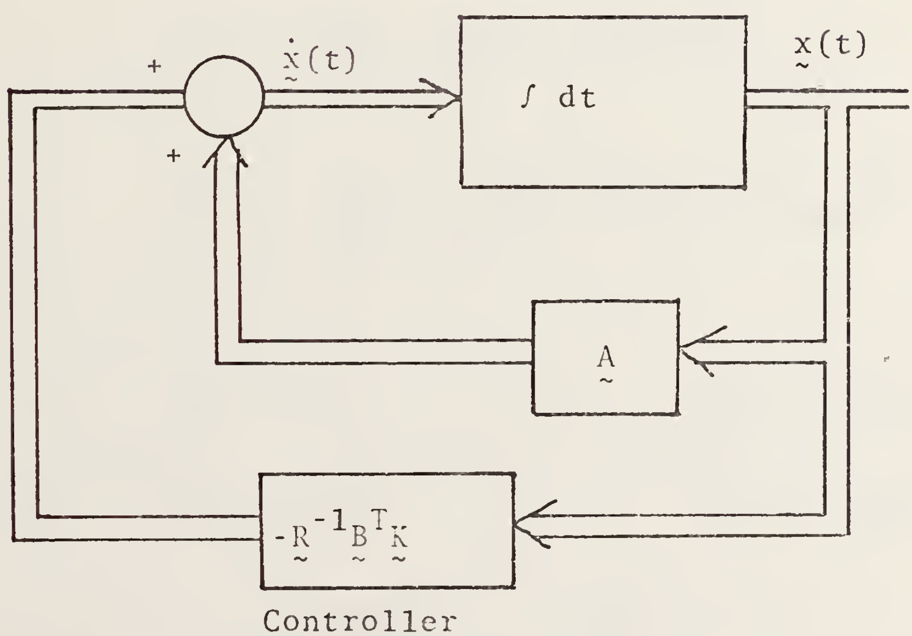


Figure (4.1)

Optimal Feedback Controller for Linear Regulator

B. APPLICATION OF A SECOND-ORDER MODEL

The second-order differential equation is expressed as

$$\frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = b_0 U \tag{4.23}$$

This state equation is:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ b_0 \end{pmatrix} U \tag{4.24}$$

The elements of K are determined by solving equation (4.21). The corresponding expressions for the optimal feedback parameters are obtained by using equation (4.22), and developed in Appendix B with the result.

$$k_0 = -\frac{a_0}{b_0} + \frac{1}{b_0} [a_0^2 + \frac{q_1}{r} b_0^2]^{\frac{1}{2}} \tag{4.25}$$

$$k_1 = -\frac{a_1}{b_0} + \frac{1}{b_0} [a_1^2 + \frac{q_2}{r} b_0^2 + 2b_0 k_0]^{\frac{1}{2}} \tag{4.26}$$

where

$$\underset{\sim}{Q} = \begin{pmatrix} q_1 & 0 \\ 0 & q_2 \end{pmatrix}, \quad \underset{\sim}{R} = \underset{\sim}{r} \; ; \quad \underset{\sim}{K} = \begin{pmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{pmatrix}$$

and $$k_0 = \frac{b_0 k_{12}}{r}, \quad k_1 = \frac{b_0 k_{22}}{r}$$

and selection of the positive parameters corresponds to the positive definite requirement on k.

V. OPTIMUM APPROXIMATION OF NUCLEAR REACTOR SYSTEM BY A SECOND-ORDER MODEL

The nonlinear high-order system described in Chapter II, which is written below

$$\dot{\underset{\sim}{x}} = \underset{\sim}{A}\,\underset{\sim}{x} + \underset{\sim}{B}\,e \tag{5.1}$$

where $\underset{\sim}{x}(t_0) = \underset{\sim}{x}_0$,

$e = x^* - x$

and $$\underset{\sim}{y}_{(t)} = \underset{\sim}{C}^T\,\underset{\sim}{x} \tag{5.2}$$

may sometimes be approximated with sufficient accuracy by a linear model for small variations in state and control. To apply the optimal control law to the nuclear reactor system, this optimum approximation problem is the most necessary thing. This idea has been developed by Sinha and Bereznai [Ref. 9]. But in this paper, the different error criteria are used.

A. DESCRIPTION OF METHOD

Considering a discrete set of values of $\underset{\sim}{y}_{(t)}$ taken over a suitable interval of time,

$$Y = \{y_1, y_2, \ldots, y_i\} \tag{5.3}$$

where $y_i = y_{(ti)}$

This set represents discrete sample points of the response of the system, which is obtained by solving equation (5.1), using the digital computer. These results are shown in Chapter III. The continuous output response $y_{(t)}$ is sampled at sufficiently close intervals of time lest the significant information is lost. The objective is to find another output set y* in optimal fashion, associated with a second-order model described by the equations.

$$\dot{\underset{\sim}{x}}{}^* = A^* \underset{\sim}{x}{}^* + \underset{\sim}{B}{}^* \underset{\sim}{\mu} \tag{5.4}$$

$$\underset{\sim}{y}{}^* = \underset{\sim}{C}\, x^* \tag{5.5}$$

such that, for the same input, the following objective is satisfied.

A scalar error performance function J is minimized

$$J = g[\underset{\sim}{W}{}_i^T (\underset{\sim}{y}{}_i - y_i^*)] \tag{5.6}$$

which is some suitable function of the errors $(\underset{\sim}{y}{}_i - \underset{\sim}{y}{}_i^*)$ with a vector weighting factor $\underset{\sim}{W}{}_i$ attached at each sampling instant.

The choice of the error criterion expressed by equation (5.6), has a direct effect on the parameters of

the approximating model. Since the purpose of the error criterion is to measure the extent to which the response of the model deviates from that of the actual system, the main problem is how to express this deviation numerically. It is normal practice to consider the norm of the output error and raise it to some power p. This objective becomes to minimize a summation of deviation.

$$J = \sum_{i} W_i \, || \underset{\sim}{y}_i - \underset{\sim}{y}_i^* ||^P \tag{5.7}$$

for the single-output case, where $\{y_i\}$ is a scalar sequence, and the weighting factor W_i is unity, equation (5.7) means a measure of the area between the curves when P=1, and the mean square error when P=2.

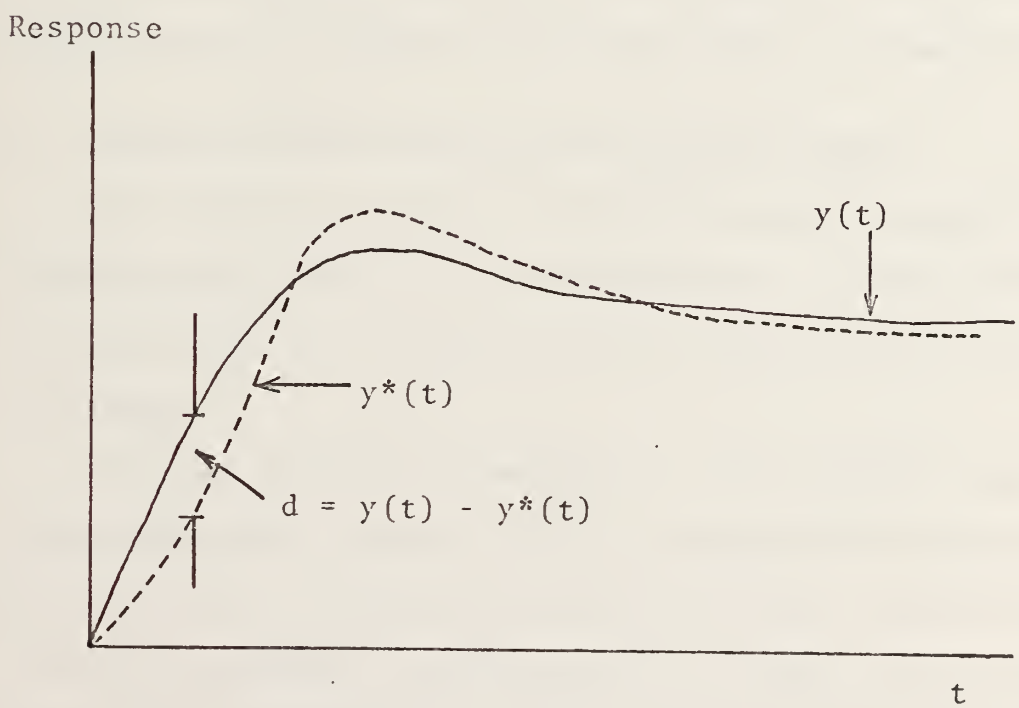


Figure (5.1)
Reference Response of the System
and the Optimal Second-Order Model

Consider the system response $y_{(t)}$ and the model response $y^*_{(t)}$ as shown in Figure (5.1). It is required to find a model of the system so that the error criterion is minimized. The following two error criteria as the performance measure are used to approximate the response $y_{(t)}$ to the response $y^*_{(t)}$ of some model.

(1) minimization of

$$J_1 = \sum_i (y_i - y_i^*)^2 \tag{5.8}$$

(2) minimization of

$$J_2 = \sum_i \{(y_i - y_i^*)^2 + (\frac{dy_i}{dt} - \frac{dy_i^*}{dt})^2\} \tag{5.9}$$

by considering the derivative of y_i and y_i^*, the more significant measure is attained where the slope is changing rapidly.

B. OPTIMUM APPROXIMATION USING PATTERN SEARCH

The problem of approximating high order system by a second-order model in an optimum manner can only be solved for specific error criteria by the presently available techniques.

Since an analytical solution to this problem does not appear feasible, various search techniques may be considered. While gradient methods are generally quite efficient in finding a minimum, the necessity of finding the partial derivatives of arbitrary error criteria with respect to all

the model parameters, becomes a great disadvantage. On the other hand, direct search techniques involve evaluating the effect of sequential parameter changes in an organized manner. The pattern-search technique of Hooke and Jeeves [Refs. 10 and 11] was selected as a suitable method for this task. Appendix C shows an outline of the pattern-search technique, which is contained within the IBM 360/67 library under the subroutine name of DIRECT.

C. INITIAL GUESS OF COEFFICIENTS OF A SECOND-ORDER MODEL

An important problem associated with search technique is the selection of the starting parameters, since these have considerable influence on the convergence of the process, and on the probability of locating a local minimum of a performance measure. Therefore, the reasonable initial guess to compute the parameters using subroutine DIRECT is needed. The characteristics of the second-order linear system have been developed in many manners [Ref. 12].

Since the response of a nuclear reactor system is approximated by a second-order model, the characteristic of that is applicable to determine the coefficients. The differential equation of a second-order system in the form of the natural frequency, W_n, and the damping ratio, ζ, is expressed as

$$\begin{aligned} \frac{d^2x}{dt^2} + 2\,\zeta\,W_n\,\frac{dx}{dt} + W_n^2\,x &= KU(t) \qquad (5.10) \\ &= KA \end{aligned}$$

The response x(t) and its first derivative are obtained by standard methods, and are shown below.

$$x(t) = \frac{AK}{W_n^2} [1 + \frac{1}{\sqrt{1 - \zeta 2}} \varepsilon^{-\zeta W_n t} \sin (W_n \sqrt{1 - \zeta^2} t - \psi] \qquad (5.11)$$

$$\dot{x}(t) = \frac{AK}{W_n} \frac{\varepsilon^{-\zeta W_n t}}{\sqrt{1 - \zeta^2}} \sin (W_n \sqrt{1 - \zeta^2} t) \qquad (5.12)$$

Figure (5.2) is a plot of x(t) and $\dot{x}(t)$. Since the input is a unit step, the value of A equals to one. To make the steady state value of x(t) unity,

$$x(t\ ) = \frac{AK}{W_n^2} = \frac{K}{W_n^2} = 1 \qquad (5.13)$$

Therefore, if $K = W_n^2$, the steady state value of x(t) becomes unity.

The value of x_{max} may be found by a setting equation (5.12) to zero, finding t_p, and substituting this value of t_p into equation (5.11)

$$\dot{x}(t_p) = 0 = \frac{AK}{W_n} \frac{\varepsilon^{-\zeta W_n t}}{\sqrt{1 - \zeta^2}} \sin (W_n \sqrt{1 - \zeta^2} t) \qquad (5.14)$$

Then

$$W_n t_p = \frac{\pi}{\sqrt{1 - \zeta^2}} \qquad (5.15)$$

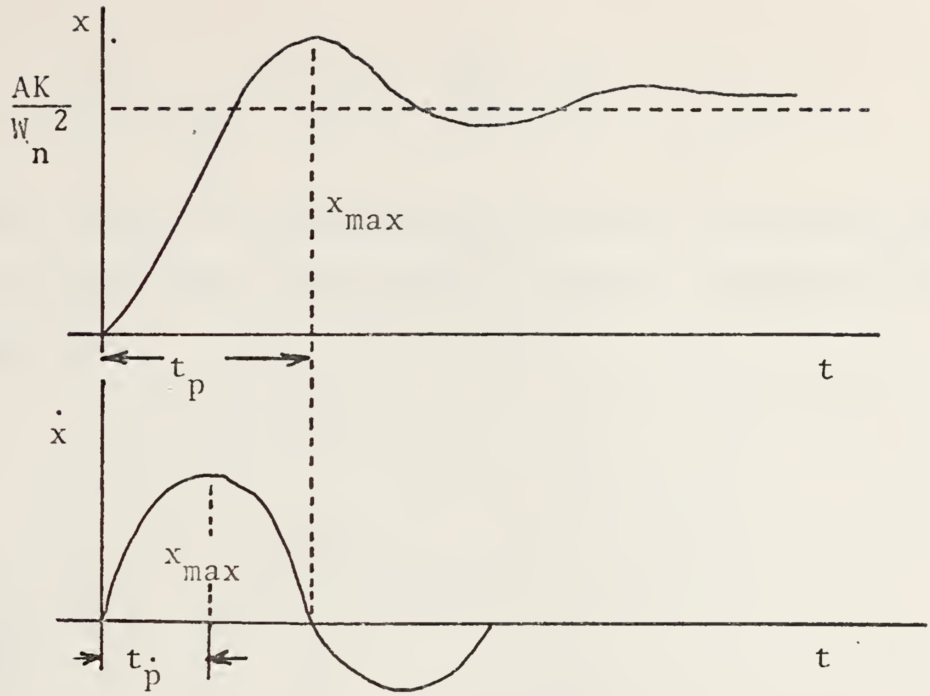


Figure (5.2)

$x(t)$ and $\dot{x}(t)$ vs. time

and

$$x(t_p) = x_{max} = 1 + \varepsilon^{-\frac{\zeta\pi}{\sqrt{1 - \zeta^2}}} \tag{5.16}$$

for a special case of a second-order system expressed in equation (4.23),

$$\dot{x}_i = x_2$$

$$\dot{x}_2 = a_0 x_1 - a_1 x_2 + b_0 U(t)$$

As stated previously, choosing $a_0 = b_0 = W_n^2$, $a_1 = 2\zeta W_n$ and $U(t)$ = unit step input, the steady state value of x_1 is

$$x_1(t\ ) = \frac{b_0}{a_0} = 1 \tag{5.18}$$

The characteristic response of the reactor system of the initial power level 90% solved by digital computer is shown in Figure (5.3)

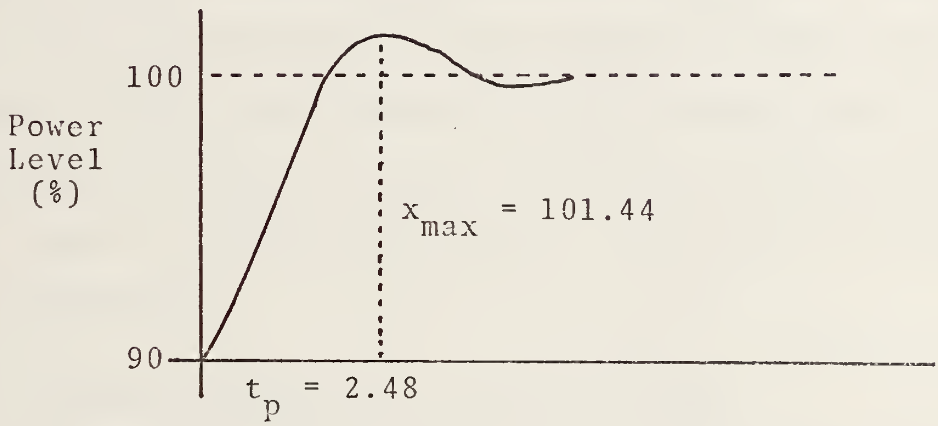


Figure (5.3)

Response of Nuclear Reactor System (90-100%)

Substituting $t_p = 2.48$ and $x_{max} = 1.0144$ into equations (5.15) and (5.16), the computed natural frequency and damping ratio are:

$$\zeta = 0.8035$$

$$W_n = 2.128$$

Then,

$$a_0 = W_n^{\ 2} = 4.528$$

$$a_1 = 2\zeta W_n = 3.42$$

Using these values as the initial guess coefficients, the computed optimal parameters by search routine are:

$$a_0 = 1.554$$

$$a_1 = 1.51$$

There is considerable difference between the initial guess values and the computed optimal coefficients. In a practical point and considering the nonlinear nature of the system, these initial guess values are sufficiently near the optimal coefficients. Since there is a definite necessity to compute initial guess values, this method is shown to be acceptable.

D. COMPUTER PROGRAM OF A PATTERN SEARCH

A computer program has been written that uses a pattern search subroutine to find optimum second-order models for high-order systems. The program has the following summarized features.

(1) The output responses of the nuclear reactor system to unit step function are available at discrete uniform intervals of time.

(2) The parameters of the model to be used for starting values are given by the user.

(3) The program assumes a uniform weighting sequence.

(4) The second-order model is solved by the Runge-Kutta-Gill fourth-order method. All calculations are in double precision.

(5) The objective functions to be minimized are equations (5.8) and (5.9).

E. MANIPULATION OF THE RESULT

Using the search routine, the coefficients of the approximated second-order model are manipulated with the results shown in Table (5.1), and the plot of that is Figure (5.4).

The difference in the model coefficients using the performance measure J_1 and that of the performance measure J_2 is small. This is clearly shown in Figure (5.5).

The system responses and the approximated second-order model of the performance measure J_2 to various initial power levels are shown in Figure (5.6) through Figure (5.11).

F. CONSIDERATION OF A THIRD-ORDER SYSTEM

Comparing the system response with the second-order model as shown in the previous part, there is some appreciable difference. But, as the considered system has a ninth-order nonlinearity, to approximate that by a second-order model, the difficulty exists in the natural sense.

So this difference is considered acceptable. As a trial, even though the optimal control law of the third-order model is not proposed, the approximated third-order model is computed using the same search routine. The equation of that is:

$$\frac{d^3x}{dt^3} + a_0 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_2 x = a_3\, U(t) \qquad (5.19)$$

Table (5.1)

Coefficients of Approximated Second-Order Model
to Various Initial Power Level

Initial Power Level		Time Interval
$N_{IC} = 0.95$	(28 sample points)	0.25
$J_1 = 8.191 \times 10^{-5}$		
$a_0 = 4.399$		
$a_1 = 2.149$		
$J_2 = 1.399 \times 10^{-3}$		
$a_0 = 4.008$		
$a_1 = 1.809$		
$N_{IC} = 0.9$	(28 sample points)	0.25
$J_1 = 6.60 \times 10^{-4}$		
$a_0 = 1.651$		
$a_1 = 1.614$		
$J_2 = 5.264 \times 10^{-3}$		
$a_0 = 1.554$		
$a_1 = 1.509$		
$N_{IC} = 0.8$	(28 sample points)	0.25
$J_1 = 3.792 \times 10^{-3}$		
$a_0 = 0.507$		
$a_1 = 0.950$		
$J_2 = 1.539 \times 10^{-2}$		
$a_0 = 0.478$		
$a_1 = 0.885$		

$N_{IC} = 0.6$ (28 sample points) 0.5

$J_1 = 1.791 \times 10^{-2}$

$a_0 = 0.14$

$a_1 = 0.511$

$J_2 = 3.529 \times 10^{-2}$

$a_0 = 0.137$

$a_1 = 0.496$

$N_{IC} = 0.5$ (28 sample points) 0.5

$J_1 = 3.705 \times 10^{-2}$

$a_0 = 0.0804$

$a_1 = 0.358$

$J_2 = 6.463 \times 10^{-2}$

$a_0 = 0.0795$

$a_1 = 0.351$

$N_{IC} = 0.3$ (28 sample points) 0.5

$J_1 = 7.424 \times 10^{-2}$

$a_0 = 2.135 \times 10^{-2}$

$a_1 = 5.689 \times 10^{-2}$

$J_2 = 1.409 \times 10^{-1}$

$a_0 = 2.282 \times 10^{-2}$

$a_1 = 7.353 \times 10^{-2}$

$N_{IC} = 0.2$ (28 sample points) 0.5

$J_1 = 6.097 \times 10^{-2}$

$a_0 = 1.564 \times 10^{-2}$

$a_1 = 6.097 \times 10^{-2}$

$J_2 = 3.560 \times 10^{-1}$

$a_0 = 1.633 \times 10^{-2}$

$a_1 = 6.966 \times 10^{-2}$

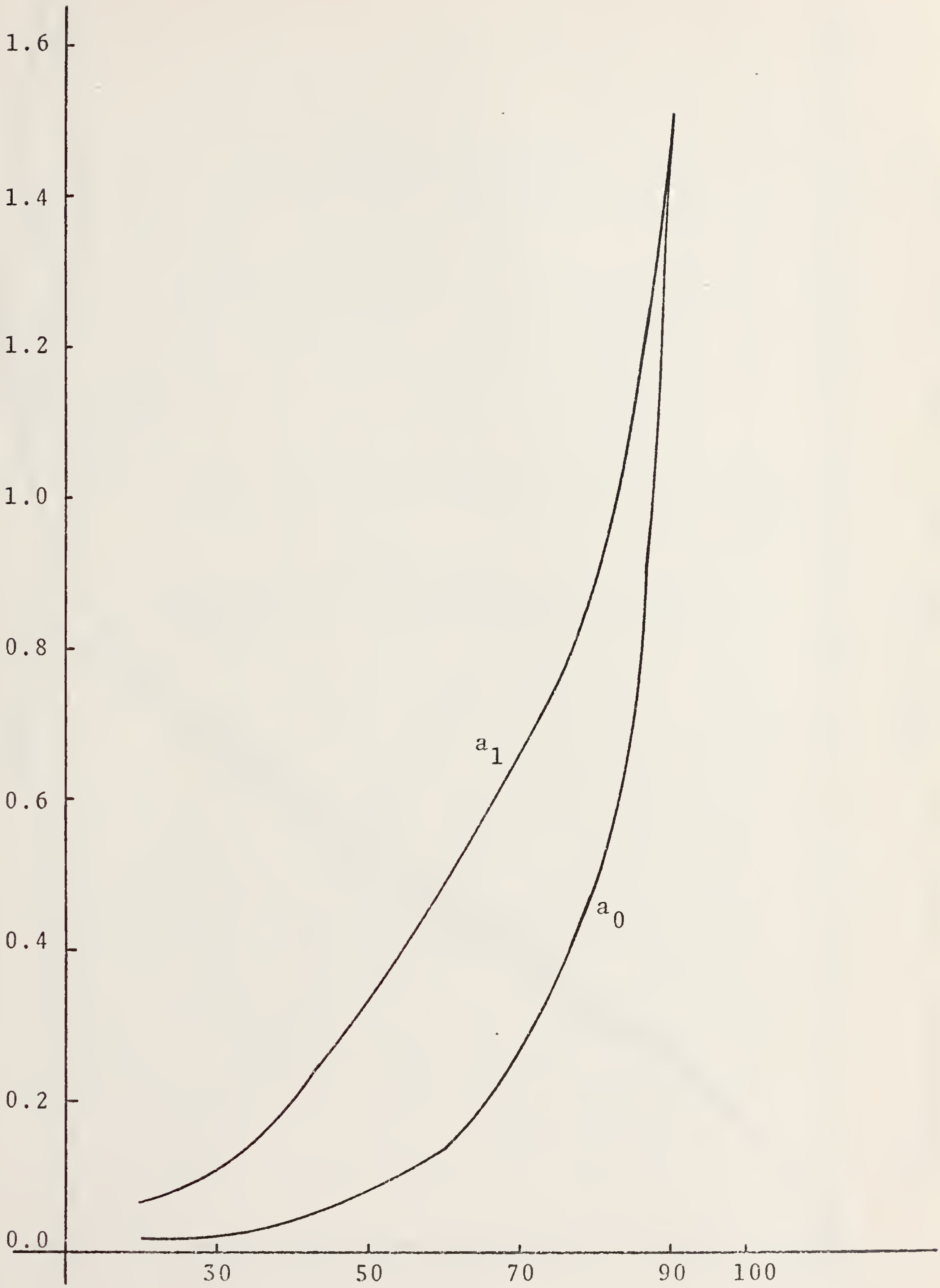


Figure (5.4)
Coefficients Plot of Second Order Model

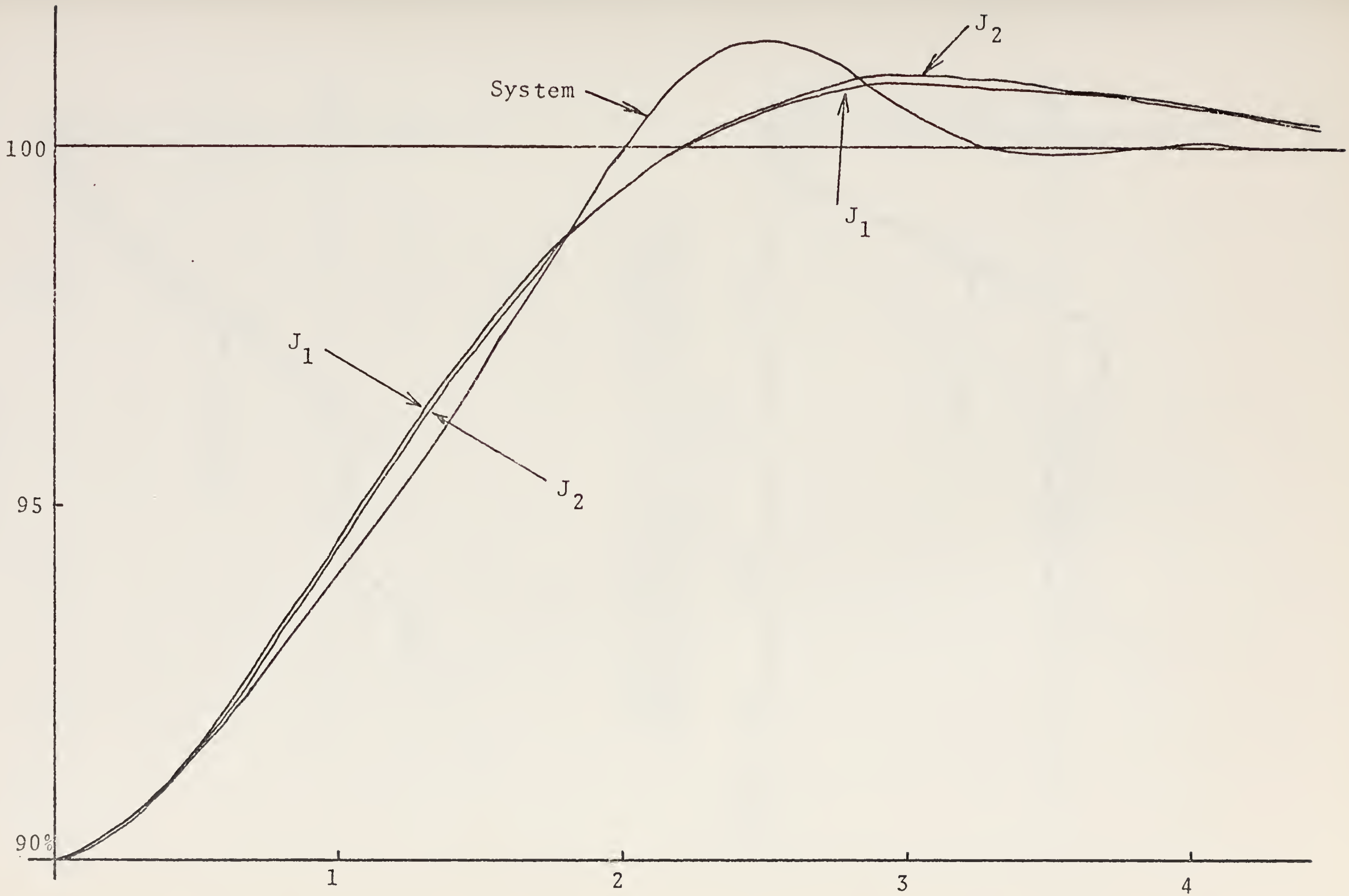


Figure (5.5)
Comparison of a System and a Second-Order Model to 90% Initial Power

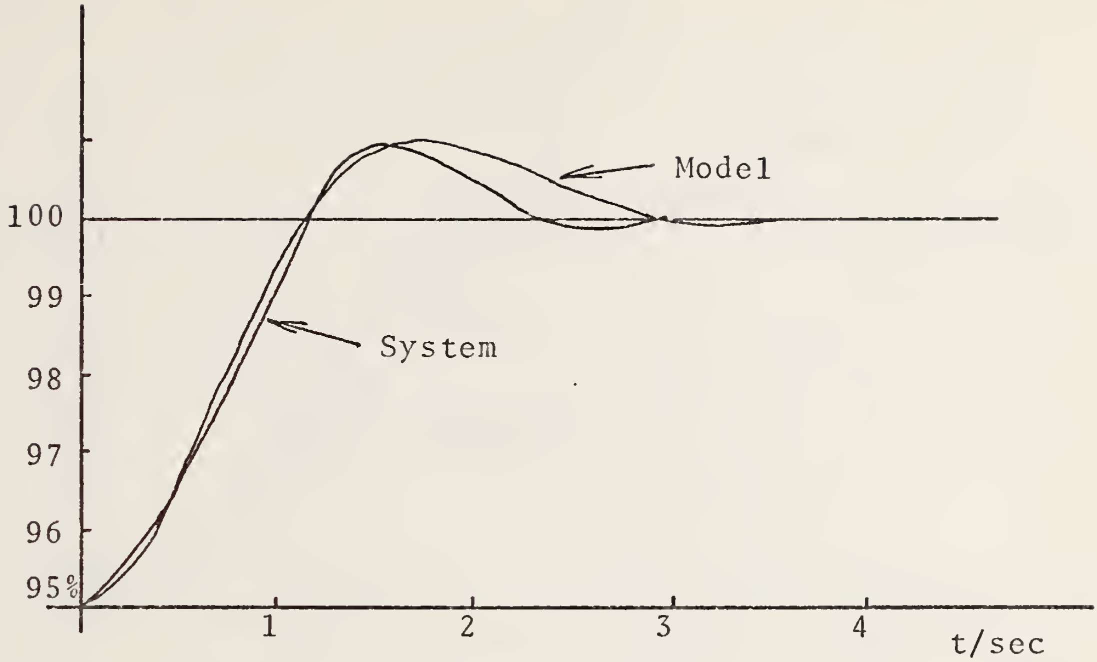


Figure (5.6)

Comparison of Step Response of System and Model to 95% Initial Power Level

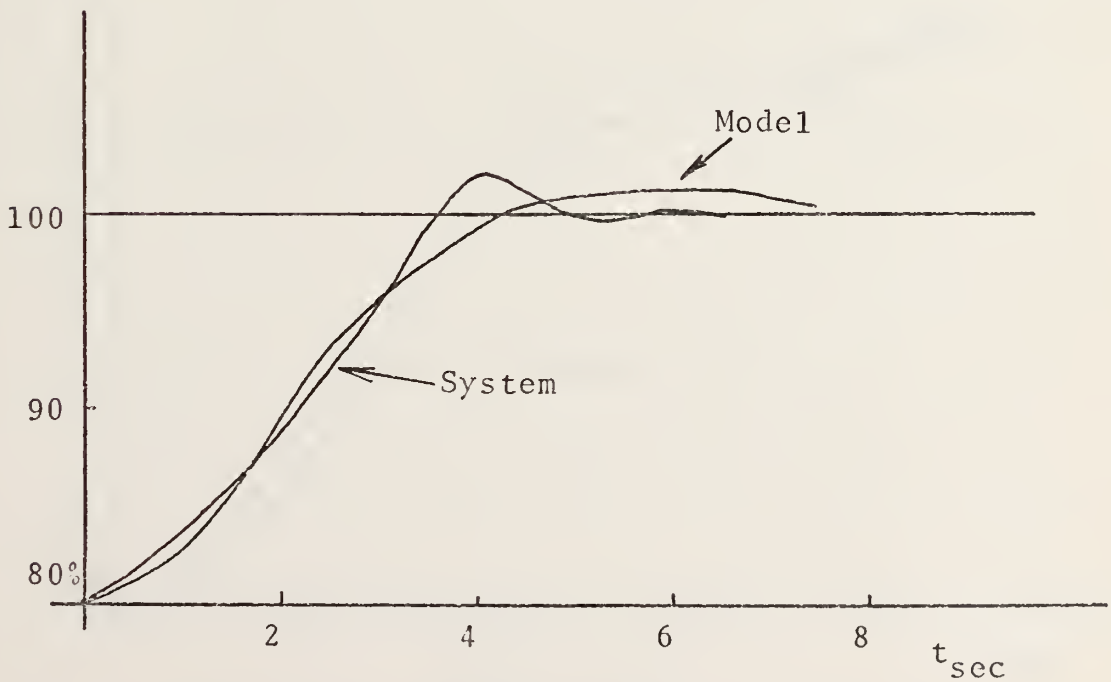


Figure (5.7)

Comparison of Step Response of System and Model to Initial Power Level 80%

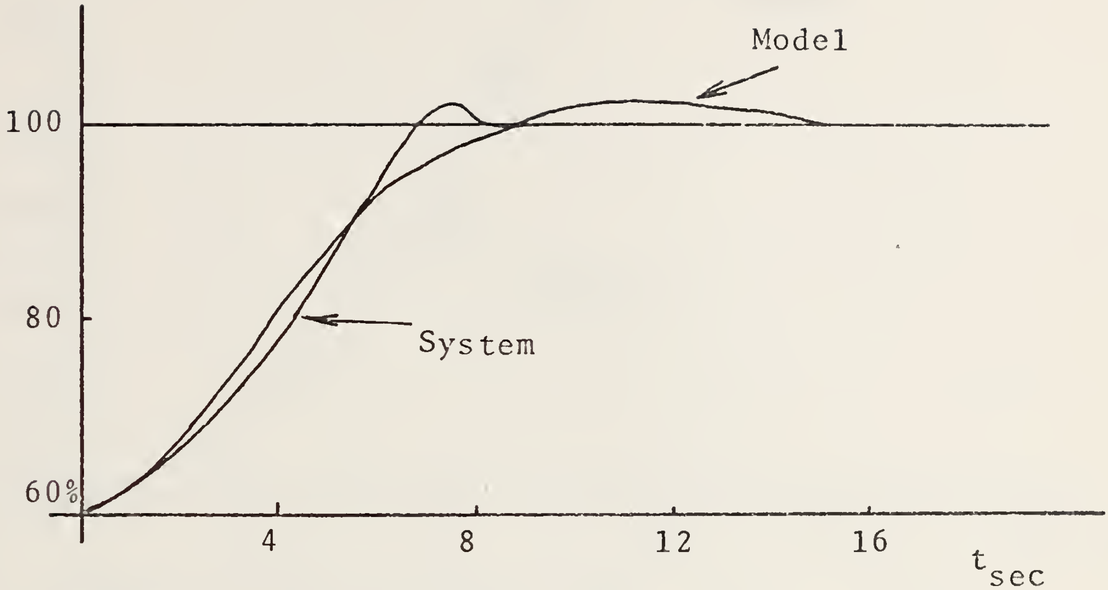


Figure (5.8)

Comparison of Step Response of System and Model to Initial Power Level 60%

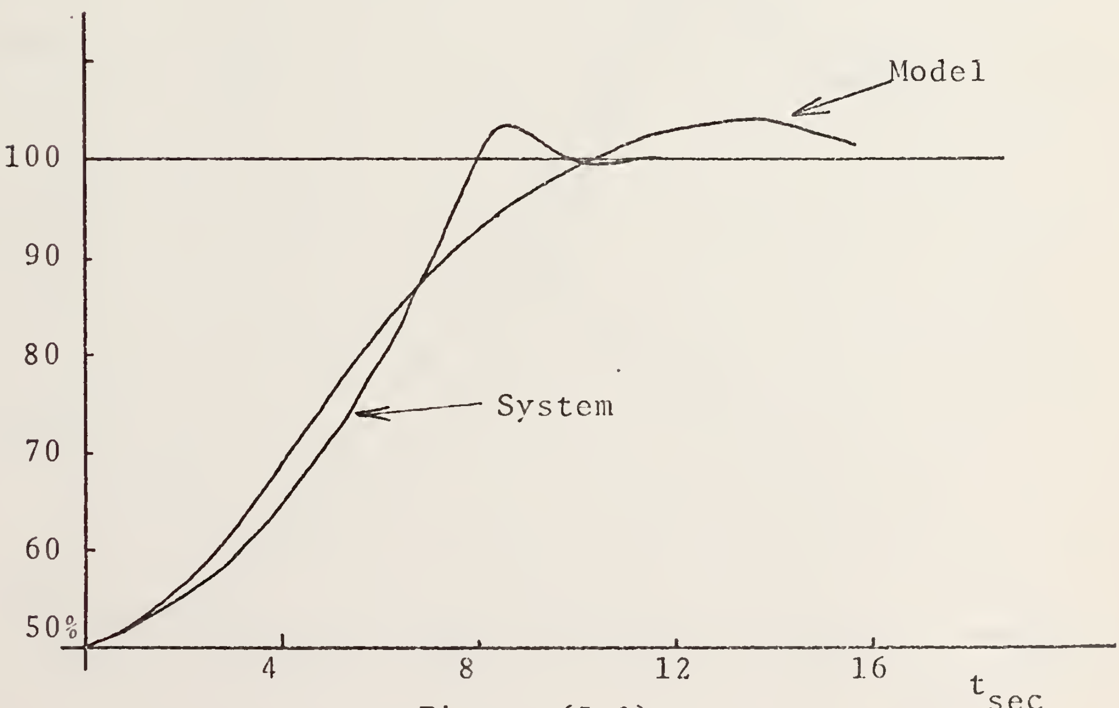


Figure (5.9)

Comparison of Step Response of System and Model to Initial Power Level 50%

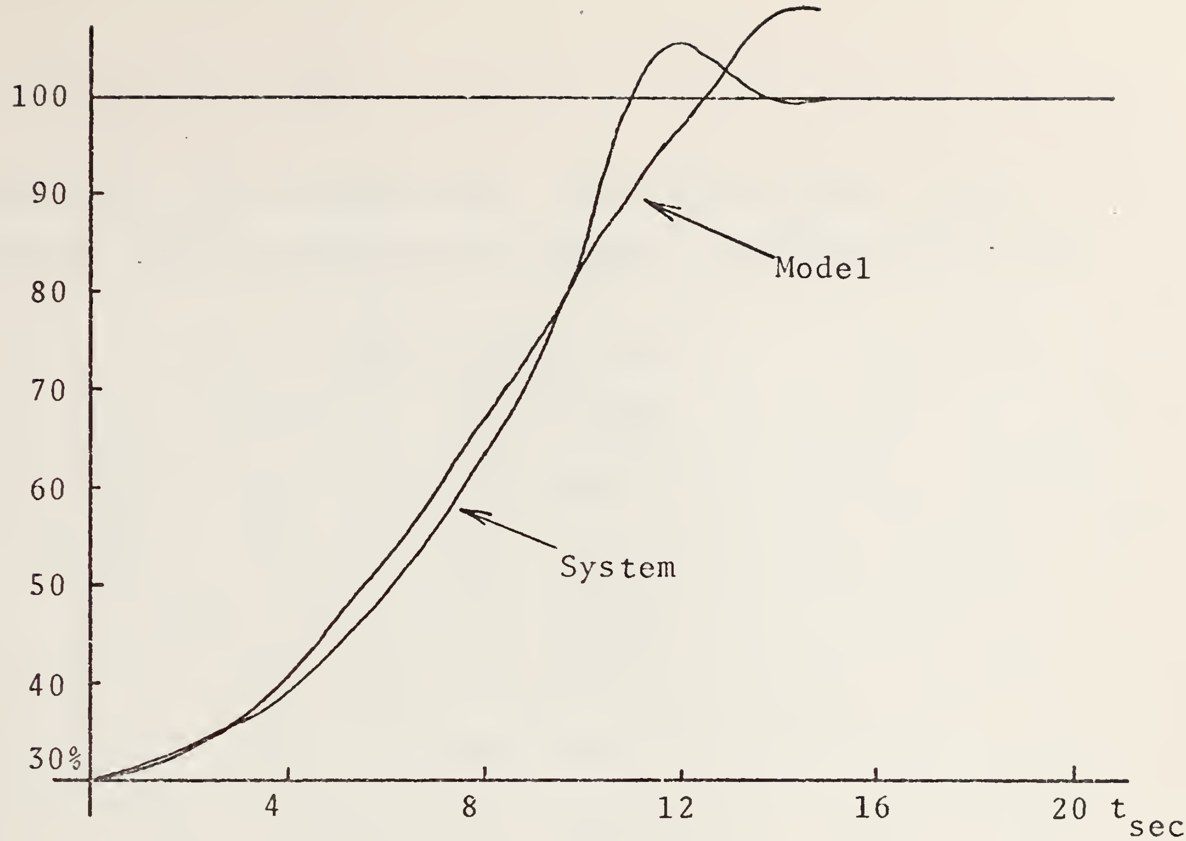


Figure (5.10)
Response to Initial Power 30%

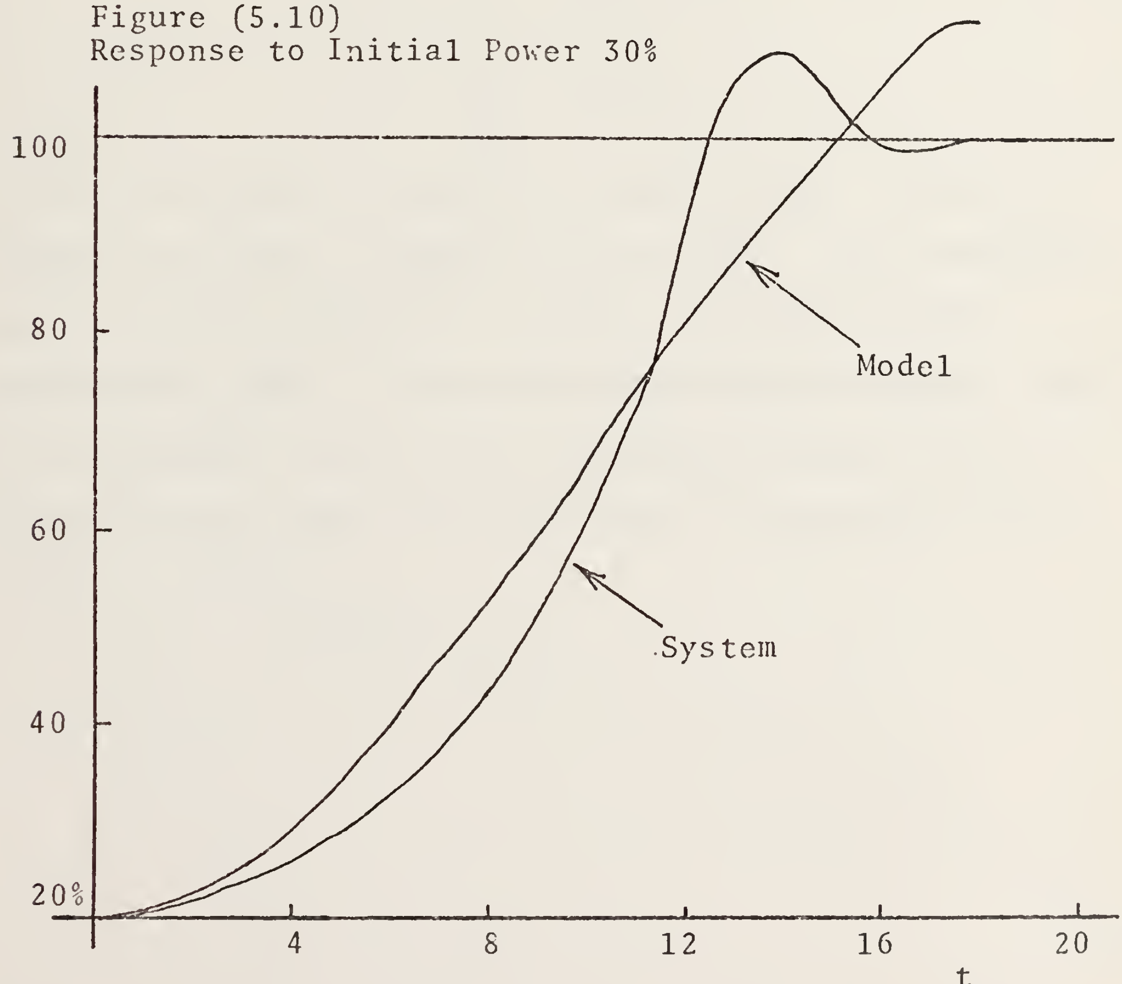


Figure (5.11)
Comparison of Step Response of System and Model to Initial Power Level 20%

where $U(t)$ = unit step input. The obtained data of the optimal third-order system to initial power level 50% is:

$$J_1 = 6.213 \times 10^{-3}$$

$$a_0 = 2.336$$

$$a_1 = 8.10$$

$$a_2 = 10.5$$

$$a_3 = 10.64$$

$$J_2 = 7.7 \times 10^{-1}$$

$$a_0 = 2.16$$

$$a_1 = 8.53$$

$$a_2 = 10.63$$

$$a_3 = 10.88$$

The clarified plot is shown in Figure (5.12). Also, the second-order model is added in this graph. It is observed that the third-order model is more realistic than the second-order model to approximate the actual system. Since it is desirable to construct the optimal control law of the third-order model, that work will be expected in the future.

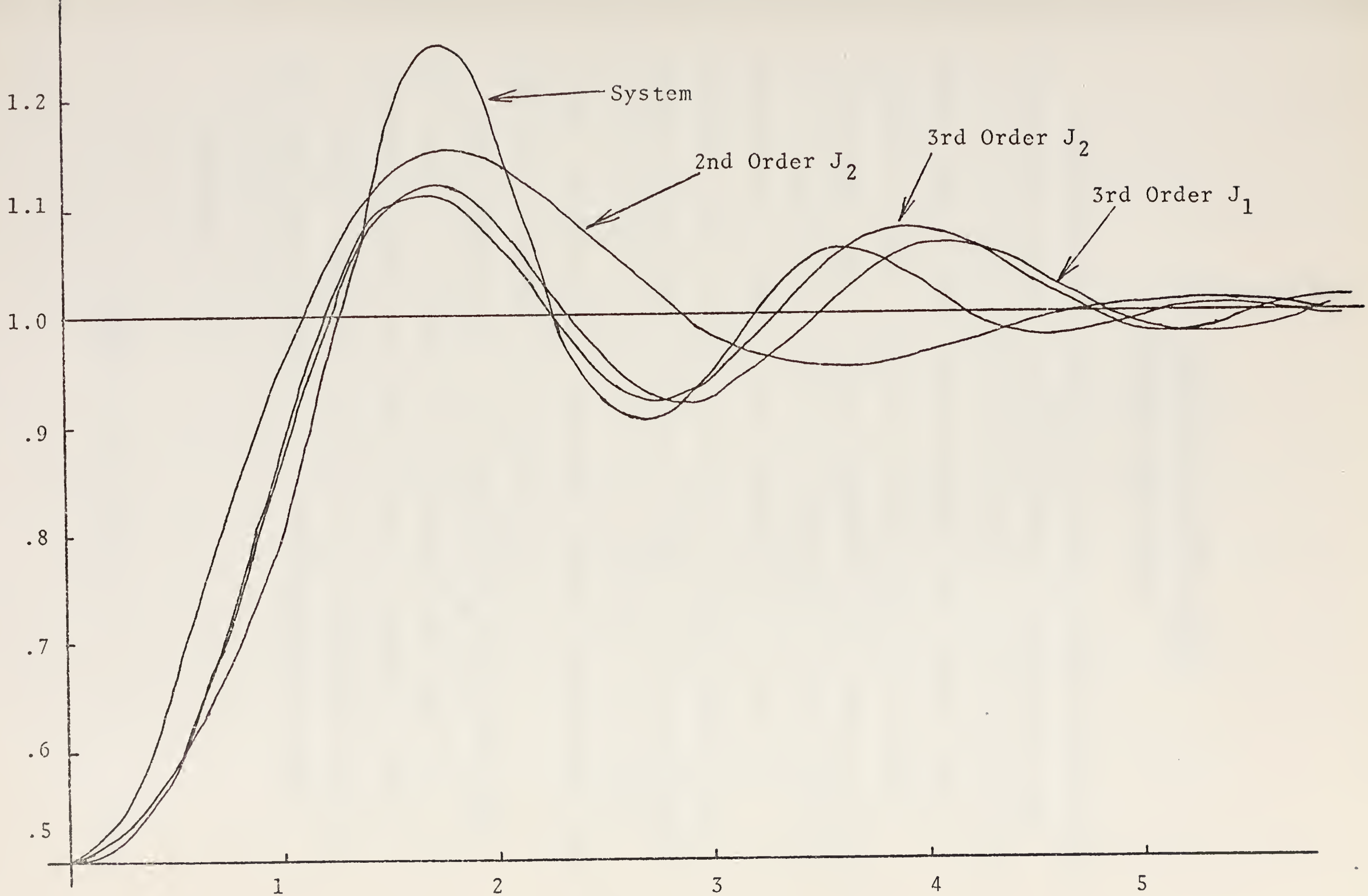


Figure (5.12)
Approximated Third-Order Model to System Response of Initial Power Level 50%

VI. SUBOPTIMAL CONTROL USING A SECOND ORDER MODEL

The suboptimal control of a reactor system using optimal second-order model is made possible by the fact that the state variables available for feedback are the actual output power level and its rate of change. The optimum linear feedback of these variables for the case of the second-order model with an integral quadratic cost function exists, and the appropriate feedback coefficients can be determined explicitly in terms of the parameters a_0, a_1 and the constants used in the cost function. These realize a suboptimal control for the reactor system. The block diagram representation of the suboptimal controller, the reactor system and the optimal controller for the second-order model are shown in Figures (6.1) and (6.2).

Using Table (5.1), the calculated k_0 and k_1 are shown in Table (6.1). Hence, the weighting factor $q_1 = q_2 = r$ in equations (4.25) and (4.26) is used to compute k_0 and k_1.

In this system and model with the suboptimal controller and the optimal controller, one modification is needed to make the steady state value unity.

From Figure (6.2)

$$\frac{x_1(s)}{\mu(s)} = \frac{a_0}{s^2 + a_1 s + a_0} \qquad (6.1)$$

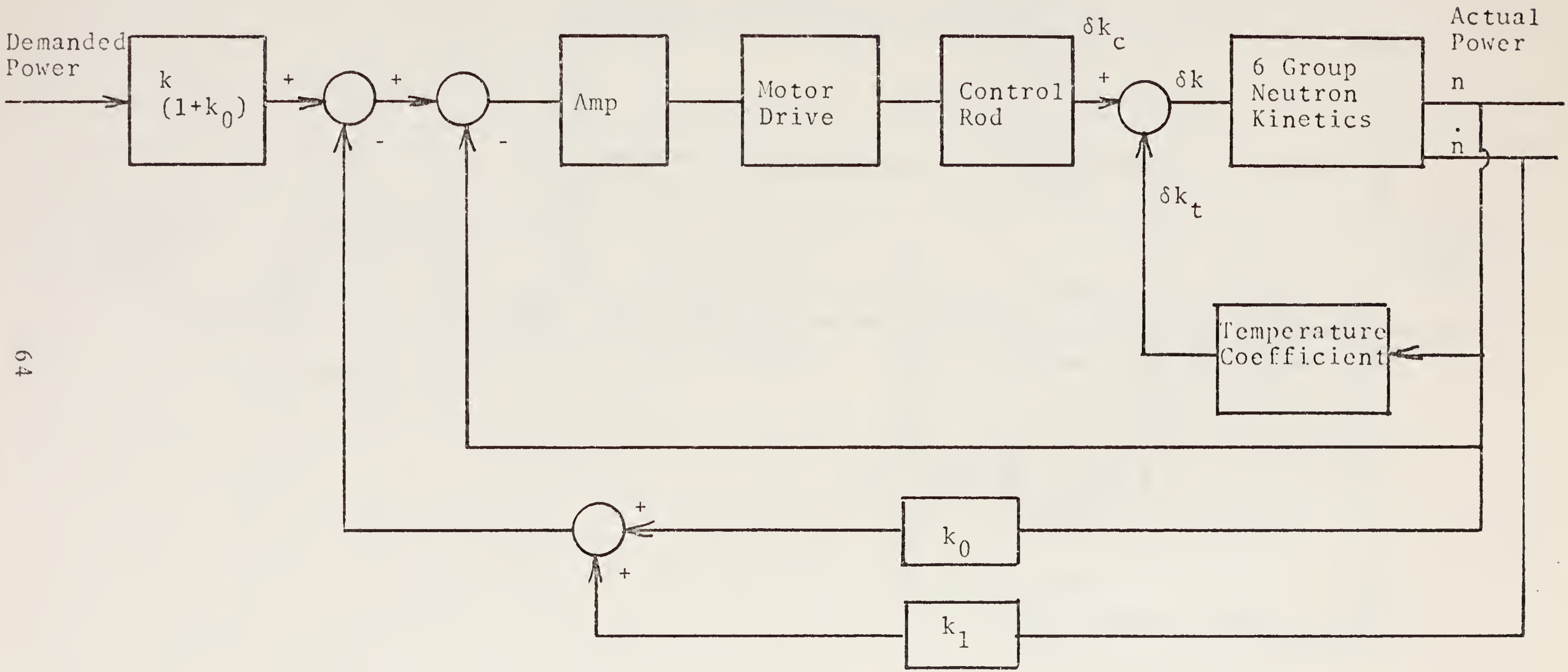


Figure (6.1)

Reactor and Suboptimal Controller for Step Demand Changes

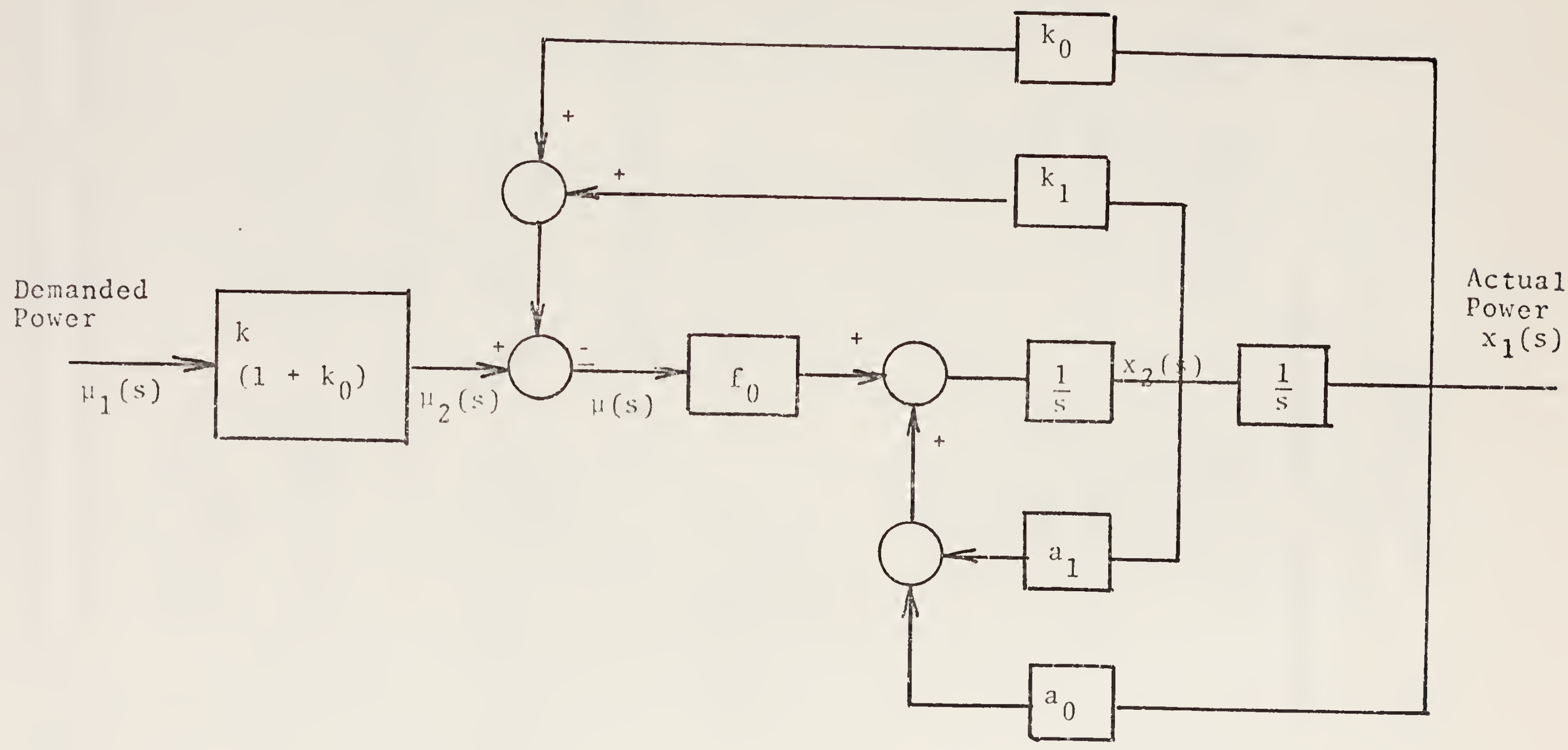


Figure (6.2)
Second-Order Model and Optimal Controller for Step Demand Changes

Table (6.1)

Controller Parameters

to Various Initial Power Level

Initial Power Level	k_0	k_1
95%	0.4142	0.736
90%	0.4142	0.602
80%	0.4142	0.631
60%	0.4142	0.869
50%	0.4142	1.144

and

$$U(s) = k\, U_1(s) - (k_0 + k_1 s)\, x_1(s) \tag{6.2}$$

Substituting (6.2) into (6.1)

$$\frac{x_1(s)}{\mu_1(s)} = \frac{a_0 k}{s^2 + (a_1 + k_1)\, s + a_0\,(1 + k_0)} \tag{6.3}$$

Using the final value theorem

$$x_{1(\infty)} = \lim_{s \to 0} s\, x_1(s)$$

$$= \frac{a_0 k}{a_0\,(1 + k_0)} \tag{6.4}$$

Therefore, the feed forward controller of the amplitude $k = 1 + k_0$ to make the steady-state value unity is needed.

To make the comparison possible, between the responses of the suboptimal controller and the corresponding optimal model controller, and that of the system and the model without any controller, Figure (6.3) shows a plot of the system response to initial power level 90% without the controller and with the controller. The describable fact is that the maximum overshoot of the system response with the suboptimal controller is reduced. Also, Figure (6.4) shows a plot of the model response to the same initial power level without the controller and with the controller. The desirable reduction of the time to reach the 100% power level is observed in the model response with the optimal controller. So, the considered suboptimal controller works to make the reduction of the maximum overshoot of the system response with no effect on the early time response. On the other hand, the optimal controller of the model does affect the early time response by making it faster. Table (6.2) is the comparison of the overshoot and the time to reach the first full power (100%) of the system and the model, without controller and with controller.

The responses between the system and the model to various initial power level with the controllers as shown in Figure (6.5) through Figure (6.9). Comparing these with Figure (5.5) through Figure (5.9), the controllers work to effect the model rise time in such a manner to make the overshoot of the system and the model occur at the same time, and have approximately the same value.

Table (6.2)

Comparison of Some Reference Points between without Controller and with Controller

Initial	Magnitude of Overshoot			Time of First Full Power		
Power Level %	Without Controller	With Controller	Reduced Overshoot	Without Controller	With Controller	Difference
95%						
System	101.05	100.82	21.9%	1.16 sec.	1.16 sec.	0
Model	101.02	100.81		1.13	0.97	
90%						
System	101.44	100.71	50.7%	2.02	2.04	+0.02
Model	100.55	100.61		2.25	1.88	
80%						
System	102.03	101.08	46.8%	3.59	3.61	+0.02
Model	100.78	101.47		4.28	3.30	
60%						
System	103.07	102.41	21.5%	6.53	6.53	0
Model	101.08	103.25		8.46	6.08	
50%						
System	103.81	102.30	39.6%	7.99	8.02	+0.03
Model	102.42	105.14		10.23	7.59	

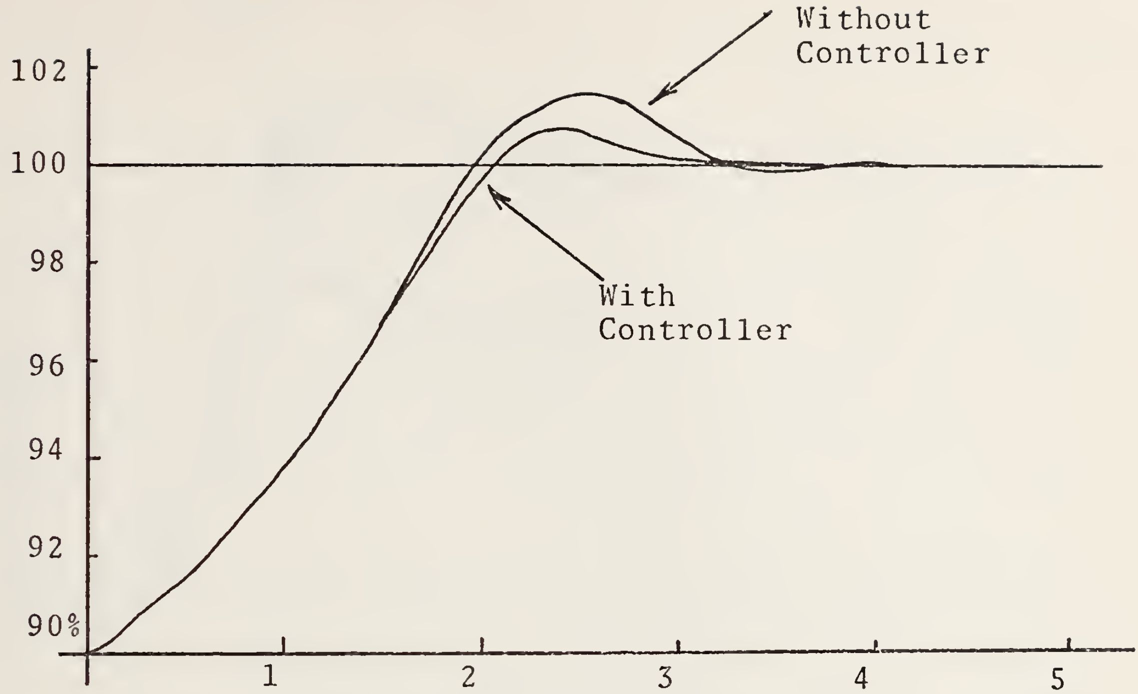


Figure (6.3)

Comparison of System Response of Initial Power Level 90% without Controller and with Controller

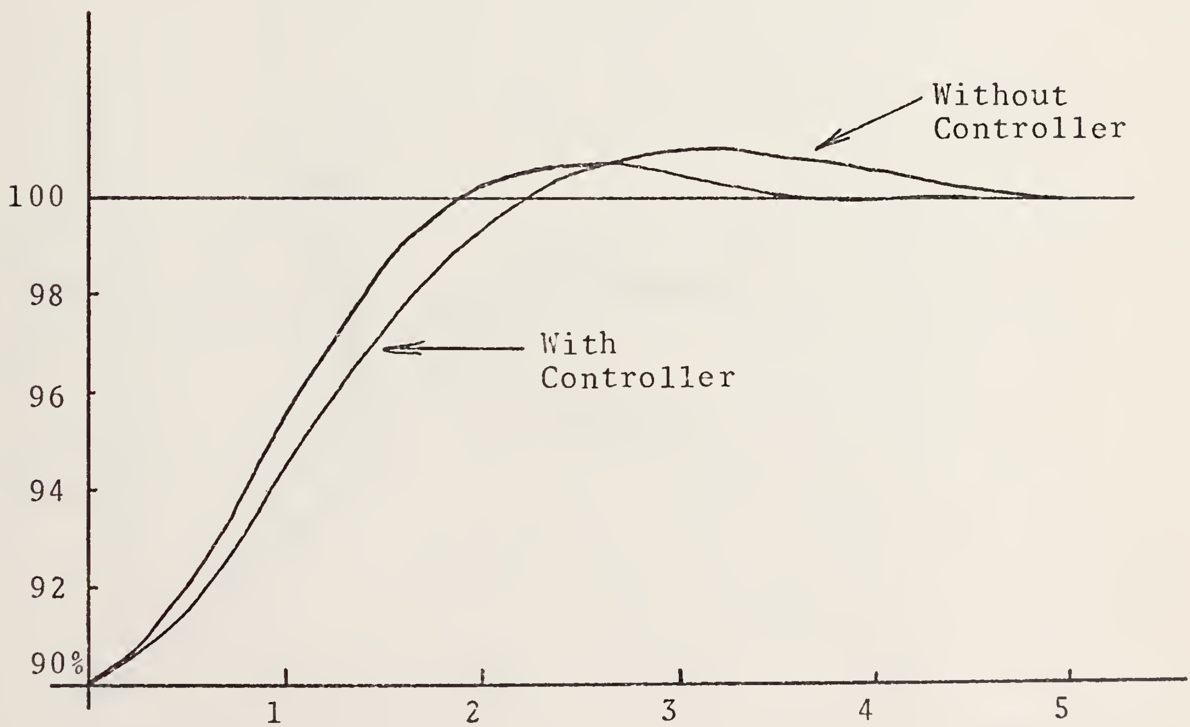


Figure (6.4)

Comparison of Model Response of Initial Power Level 90% without Controller and with Controller

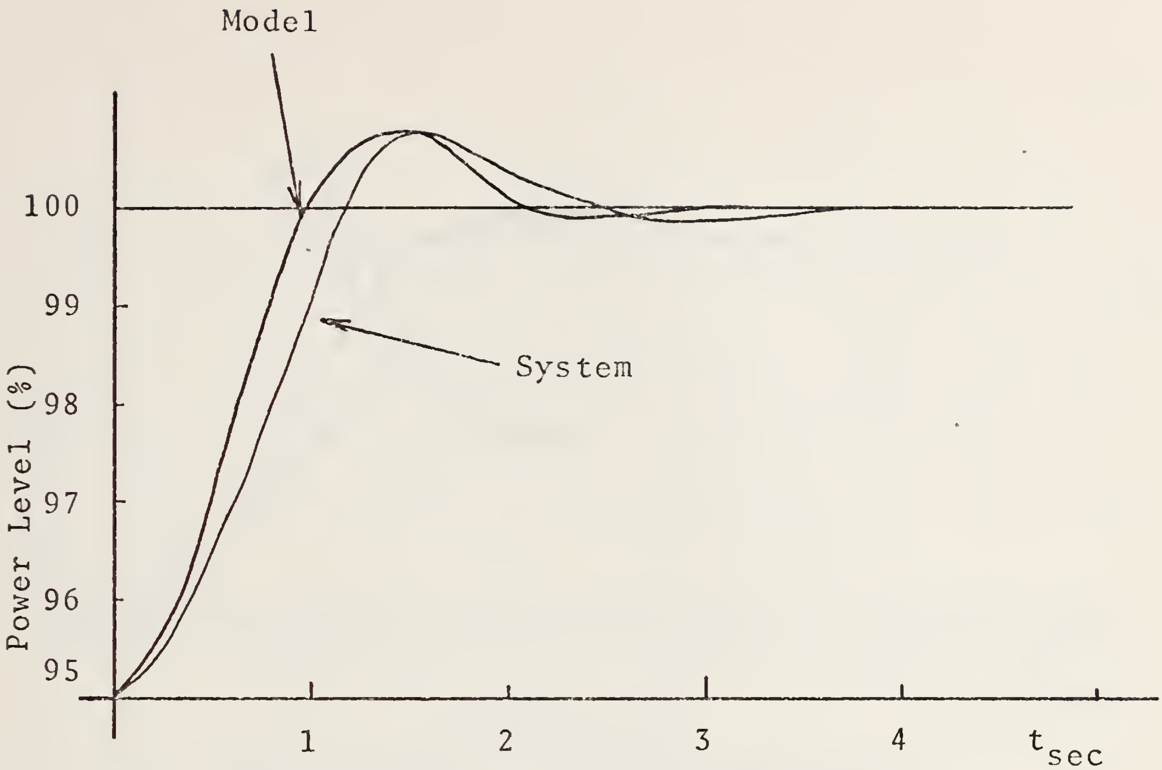


Figure (6.5)

Step Responses of System and Model
with Controllers to Initial Power Level 95%

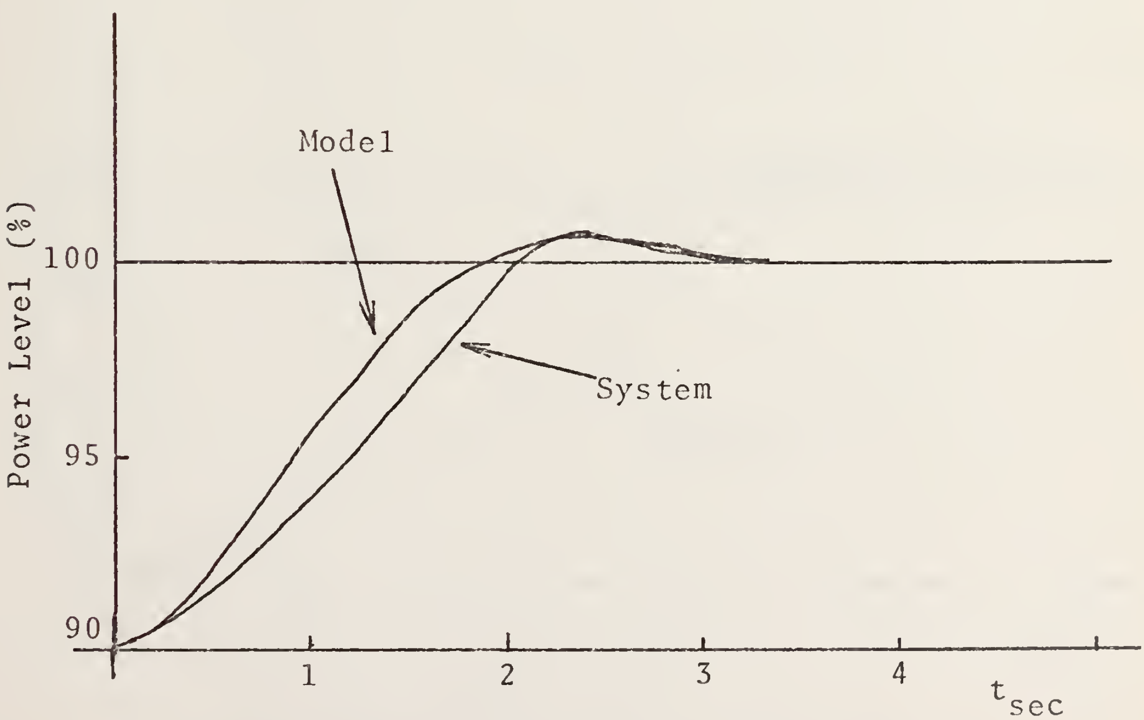


Figure (6.6)
Step Response of System and Model
with Controllers to Initial Power Level 90%

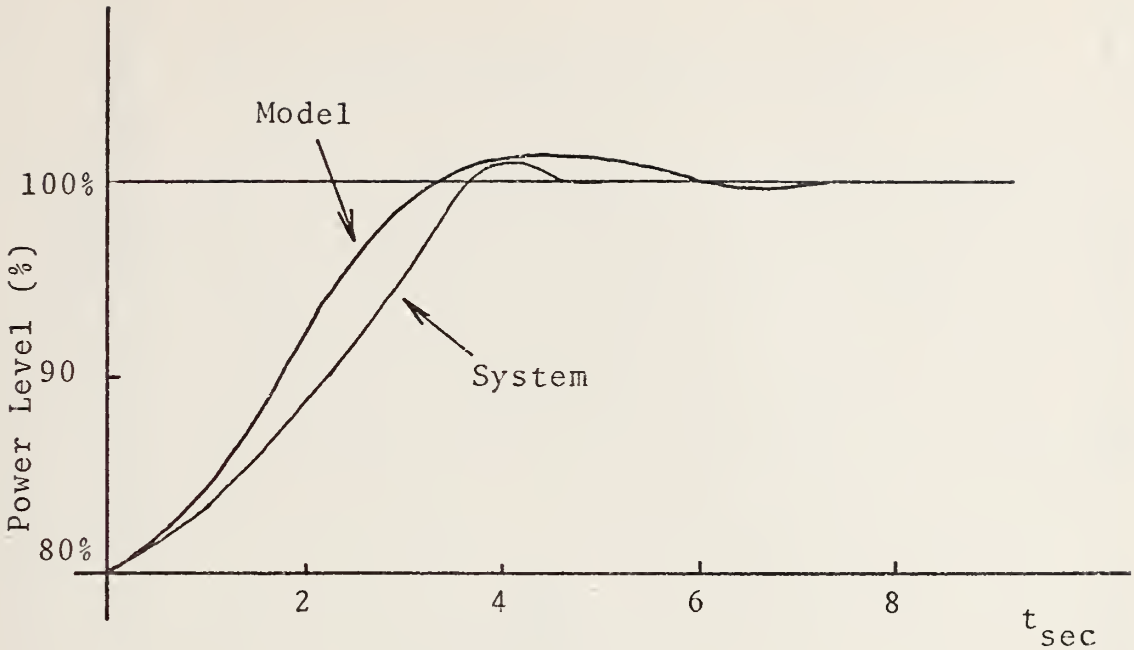


Figure (6.7)

Step Responses of System and Model
with Controllers to Initial Power Level 80%

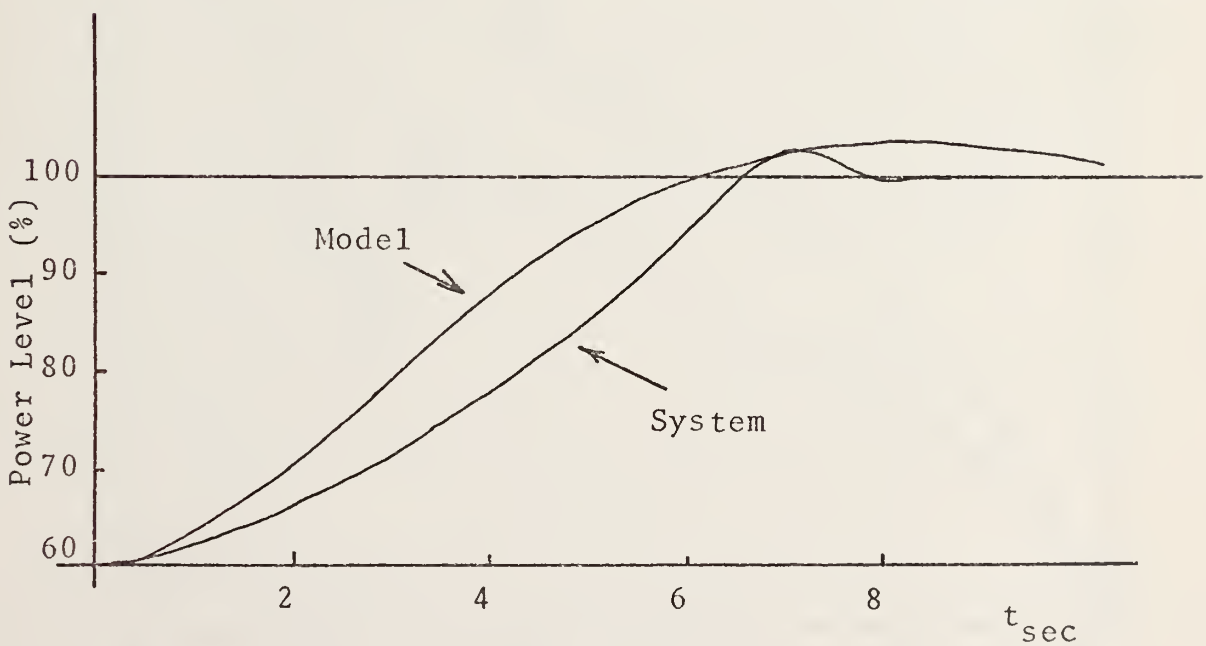


Figure (6.8)

Step Responses of System and Model
with Controllers to Initial Power Level 60%

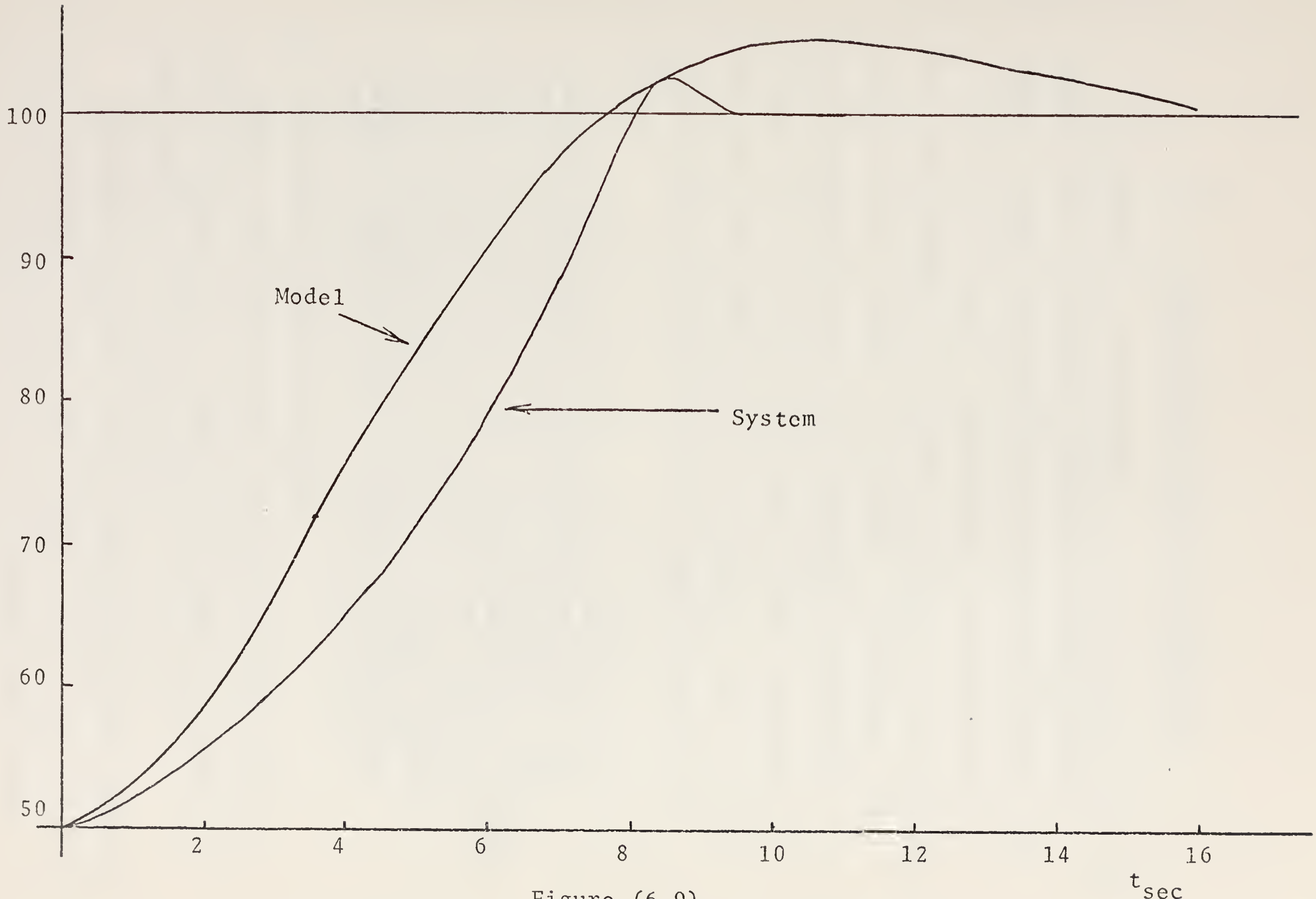


Figure (6.9)
Unit Responses of System and Model with Controllers
to Initial Power Level 50%

VII. REALIZATION OF ADAPTIVE CONTROLLER

The effect of the suboptimal reactor controller and the optimal model controller is described in the previous chapter. There the controller parameters are fixed. It is possible to design programmed time variations of controller parameters to achieve instantaneous near optimum control for the system at all times. In addition, the system parameter such as the temperature coefficient change with time and the existence of such conditions is the reason for application of an adaptive control system. Eveleigh [Ref. 13] has defined the adaptive control as follows:

> "If an index of performance (IP) is available which indicates the system's instantaneous or short-term average performance quality, and if a control loop is set up to optimize the IP automatically by adjusting controller parameters, the parameter-adjustment configuration is called an adaptive control loop. It is important to note that an adaptive controller is a parameter-adjustment loop above and beyond the normal feedback used to control position, velocity, and the like. Adaptive control is thus an effort to extend basic optimum-control concepts to time-varying systems."

A. PROCEDURE OF ADAPTIVE CONTROL

Hence, the concept of the adaptive control, which is shown in Figure (7.1) as used in Reference 2, is employed. This procedure is described below.

The model parameters are precomputed, over the expected range of demanded power level changes, to be used as starting values for the optimization. As the model coefficients

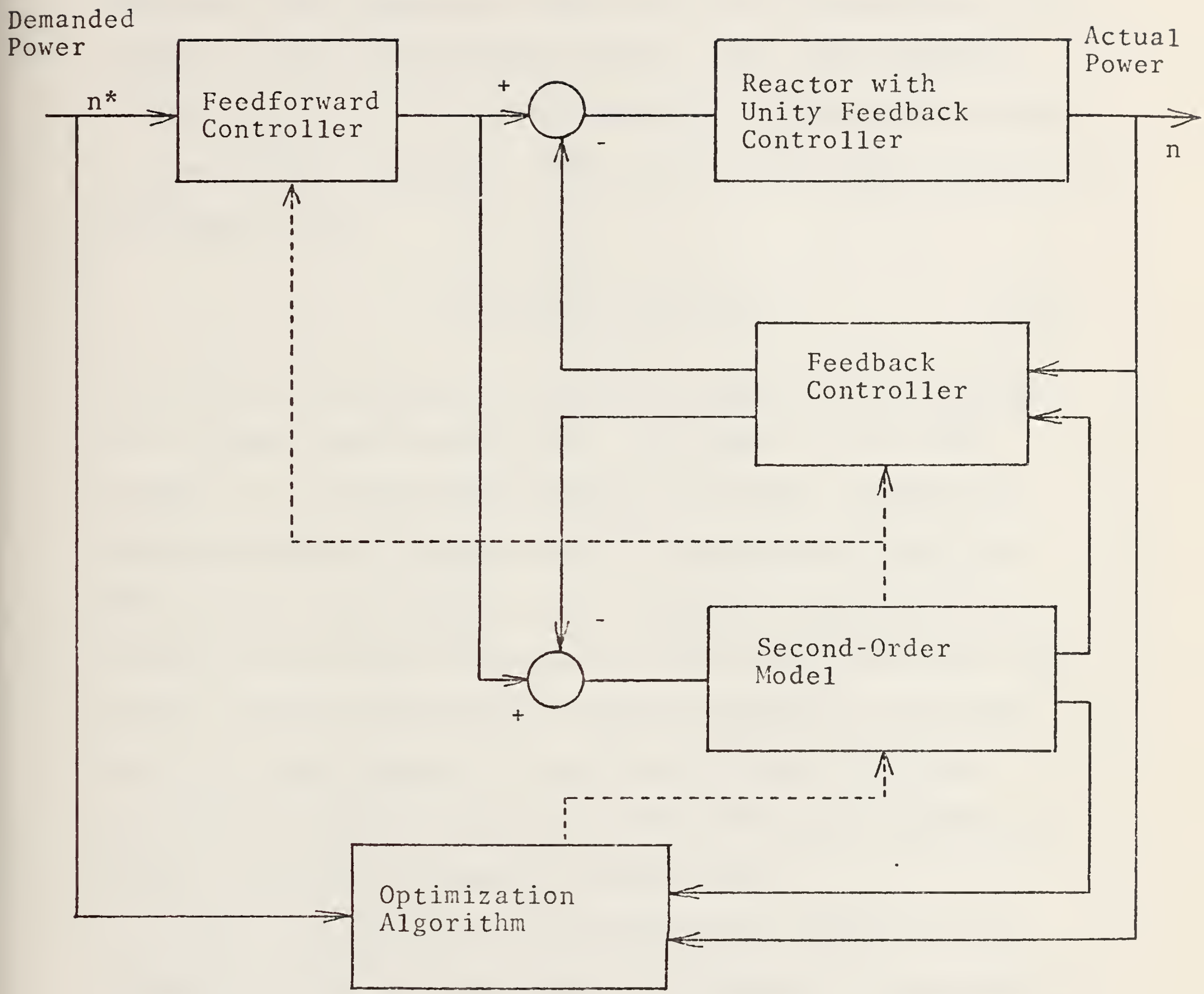


Figure (7.1)
Block Diagram of Adaptive Control System

are updated, the parameters of the feedback controller are computed from equations (4.25) and (4.26), depending on the nature of the cost function. Here, also, the weighting factor $q_1 = q_2 = r$ is used to make the comparison possible in the previous chapter. That is, the cost function has the following form:

$$J = \frac{1}{2} \int_0^\infty (\underset{\sim}{x}^2 + U^2)\, dt \tag{7.1}$$

Initially, the nuclear reactor is assumed to be operating at a steady state power level. At time $t = 0$, a step change to a new operating level is required. Using the model parameters appropriate for the demanded change and the desired cost function, the controller parameters are evaluated, and the system begins the transition to the new power level. Over several sampling intervals, the value of the system response at each sample point is stored. This means neither updating of the model nor recomputation of the controller parameters takes place.

The fitness between system and model depends on the accuracy of the starting model. The model is reoptimized on the basis of the observed system response. The next step is to recompute the controller parameters. After completing these procedures, the most optimized controller parameters of some interval is found. This optimized controller introduces the desirable system response. Repeating these steps, the system response of entire intervals is obtained in the most optimal manner.

As described in Section B of Chapter III, the high gain of the control rod rate gives the response time much influence. To make faster response, the following control rod is used instead of equation (2.5).

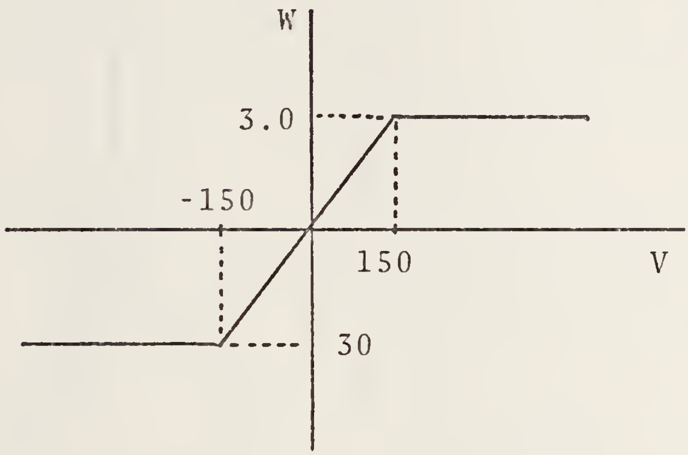


Figure (7.2)

High Gain Control Rod Characteristic

The system response without controller of initial power level 50% using the high gain control rod is shown in Figure (7.3). Also, the approximated second order model is shown in the same plot. The time to reach 100% value of the system is earlier with the higher gain rod.

Apparently, judging from this graph, the second-order model does not fit to the system response. Though the third-order model describes the system more accurately than the second-order model as shown in Figure (5.12), the usage of the third-order model is not established

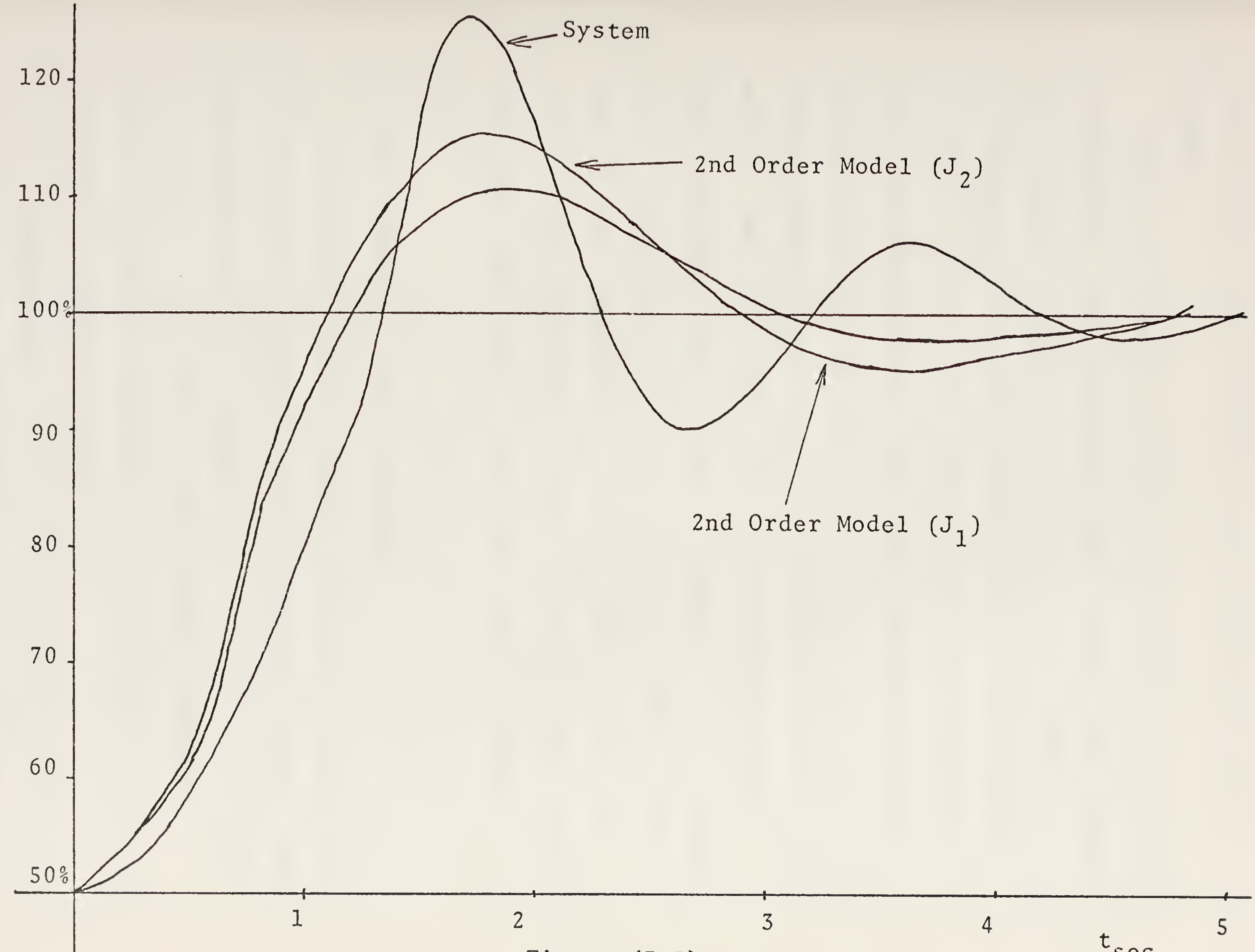


Figure (7.3)
Responses of System of High-Gain Control Rod
and Approximated Model without Controller

theoretically, and therefore is not applied to this problem.

Figure (7.4) shows a plot of the system response with the suboptimal controller to initial power level 50% and that without controller to the same initial power level. The suboptimal controller based on the optimal control law of the second-order model has the great effect to reduce the overshoot and the system response reaches steady-state value quickly. Apparently, the usage of the linear feedback controller is recommended to control the nuclear reactor to reduce the large overshoot and to reach the steady state faster. Figure (7.5) is a graph of the responses of the system and the model with the controllers. Comparing this with Figure (7.3), the second-order model with optimal controller more closely represents the reactor system than when no controller is used.

B. APPLICATION OF ADAPTIVE CONTROL

As described in the previous section, the procedures of the adaptive controller are applied by using an observation interval of 15 seconds duration and the adaption intervals of 0.25 second. The obtained parameters that are computed repeatedly until sufficient accuracy is attained, are shown in Table (7.1).

The corresponding responses of system and model with the adaptive controller are shown in Figure (7.6). Also,

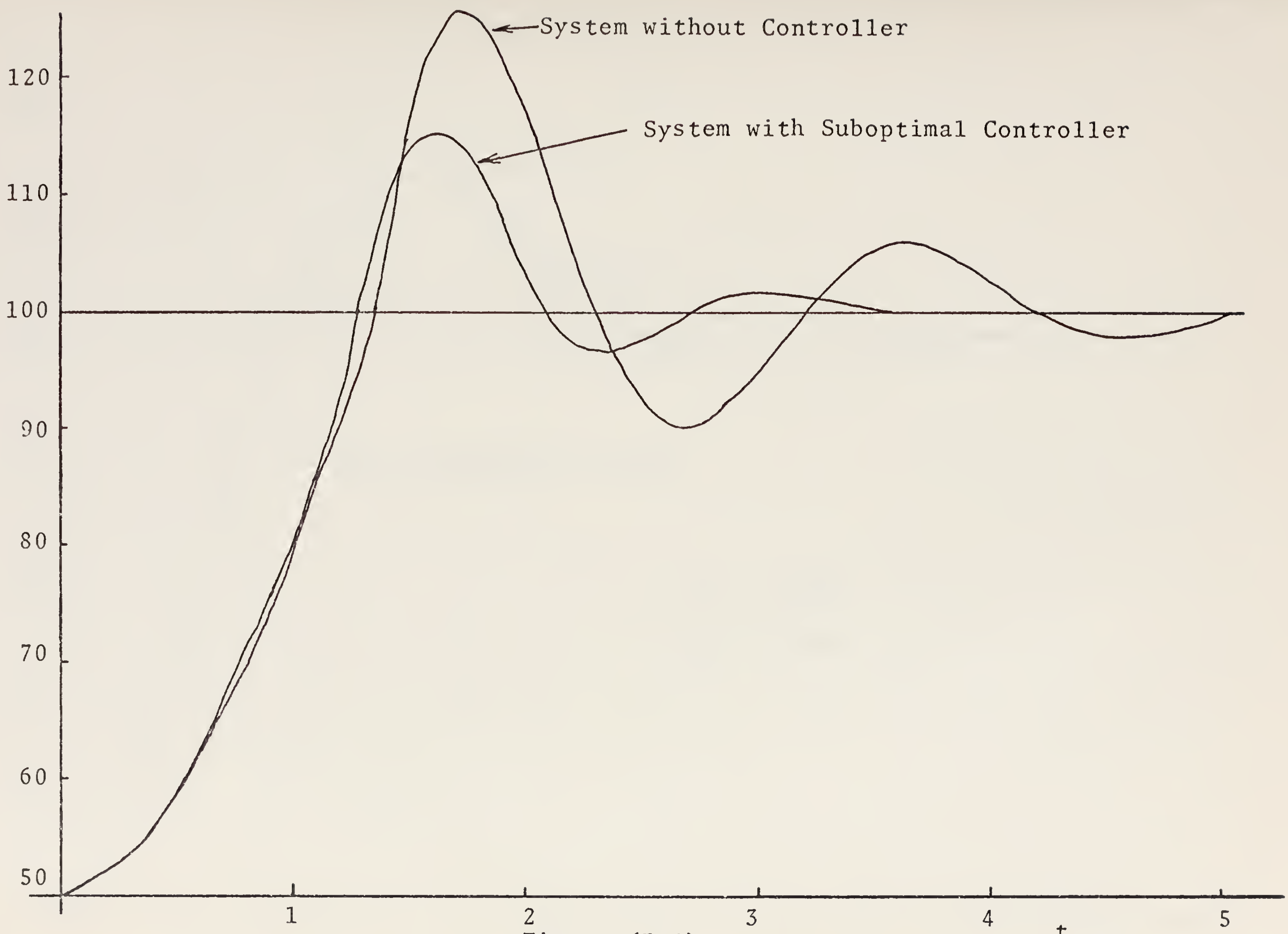


Figure (7.4)
Effect of Suboptimal Controller

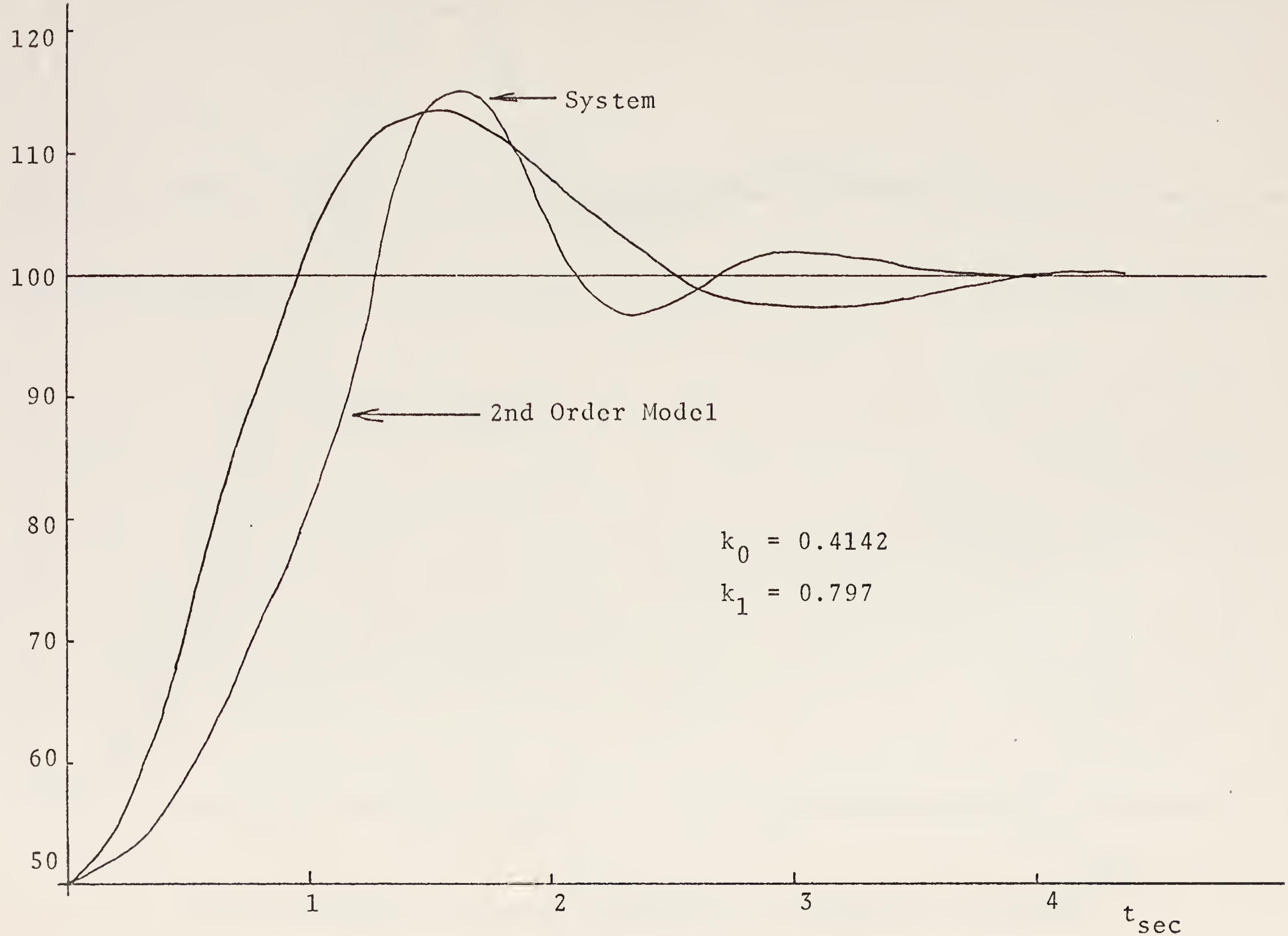


Figure (7.5)
Step Responses of System and Model with Controller

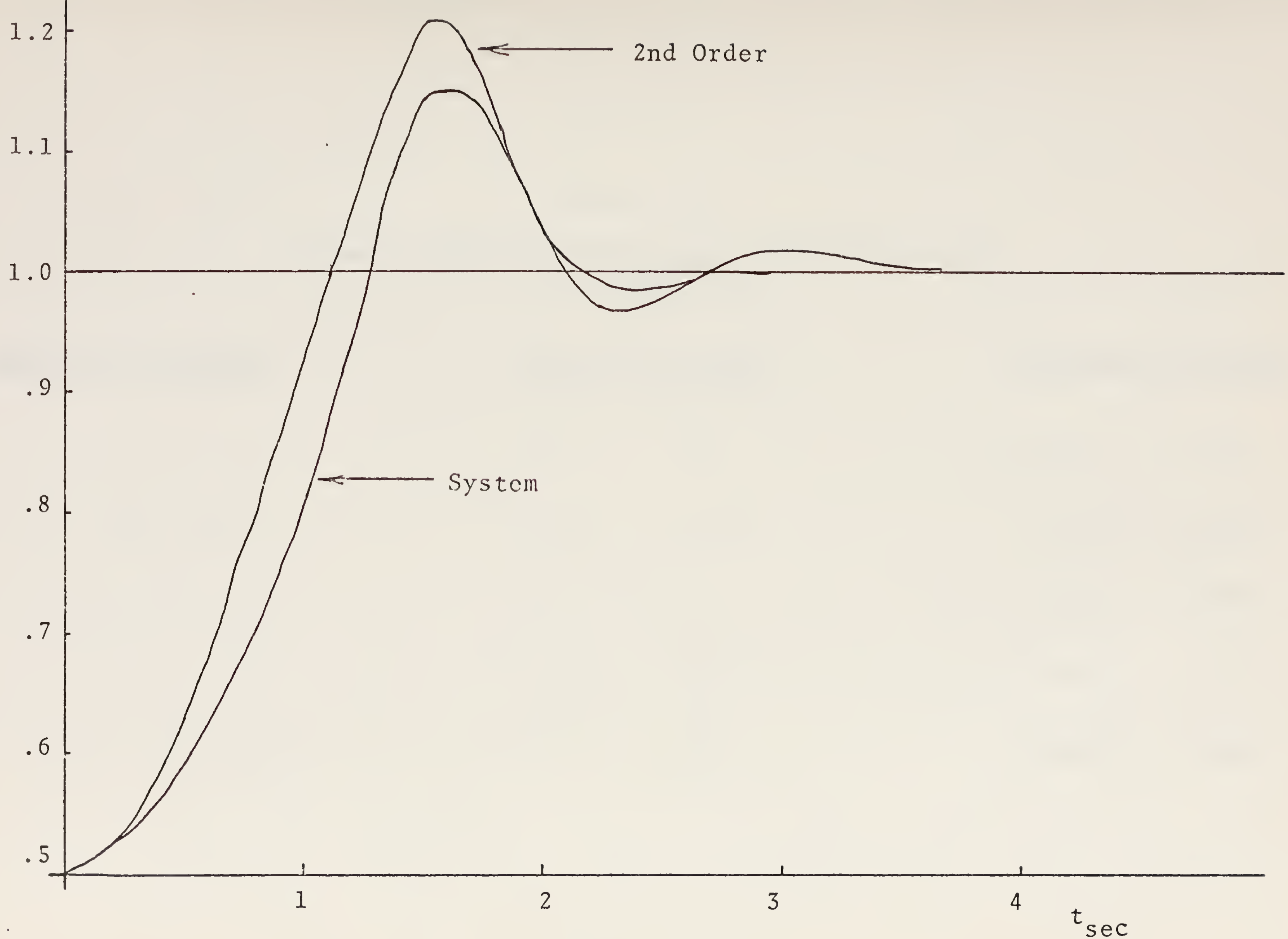


Figure (7.6)
Step Responses of System and Model with Adaptive Controller

Table (7.1)

Parameter Changes during Power Level Transitions
(50 - 100% PF)

Adaptation Interval	Model Coefficient of 50 - 100% FP		Feedback Parameter	
	a_0	a_1	k_0	k_1
1. (0 - 1.5 sec.)	1.657	0.197	0.4142	1.112
2. (1.5 - 3.0 sec.)	20.67	3.7	0.4142	0.856
3. (3.0 - 4.5 sec.)	29.32	9.16	0.4142	0.749
4 (4.5 - 6.0 sec.)	4.65	8.89	0.4142	0.287

Figure (7.7) shows the system responses with the non-adaptive controller and with the adaptive controller. A comparison of Figure (7.5) and Figure (7.6) shows that there is some improvement between the model response with the optimal controller and that with the adaptive controller. In Figure (7.7), there is no appreciable difference between the system response with the non-adaptive controller and that with the adaptive controller. This means the concept of adaptive controller is not effective as long as the system parameters are constant and unity weighting factors are used in the cost function.

As a reference, the response of the second-order model used to develop the adaptive controller is shown in Figure (7.8) with the system response using the adaptive controller.

To show the effect of adaptive controller clearly, another example is studied. The theory of adaptive control is applied to the system of initial power level 80% shown in Figure (7.9). The tabulated result is shown in Table (7.2).

Figure (7.9) is a plot of the system responses with the non-adaptive controller, and with the adaptive controller. Though there is very little difference between them, they are almost considered identical.

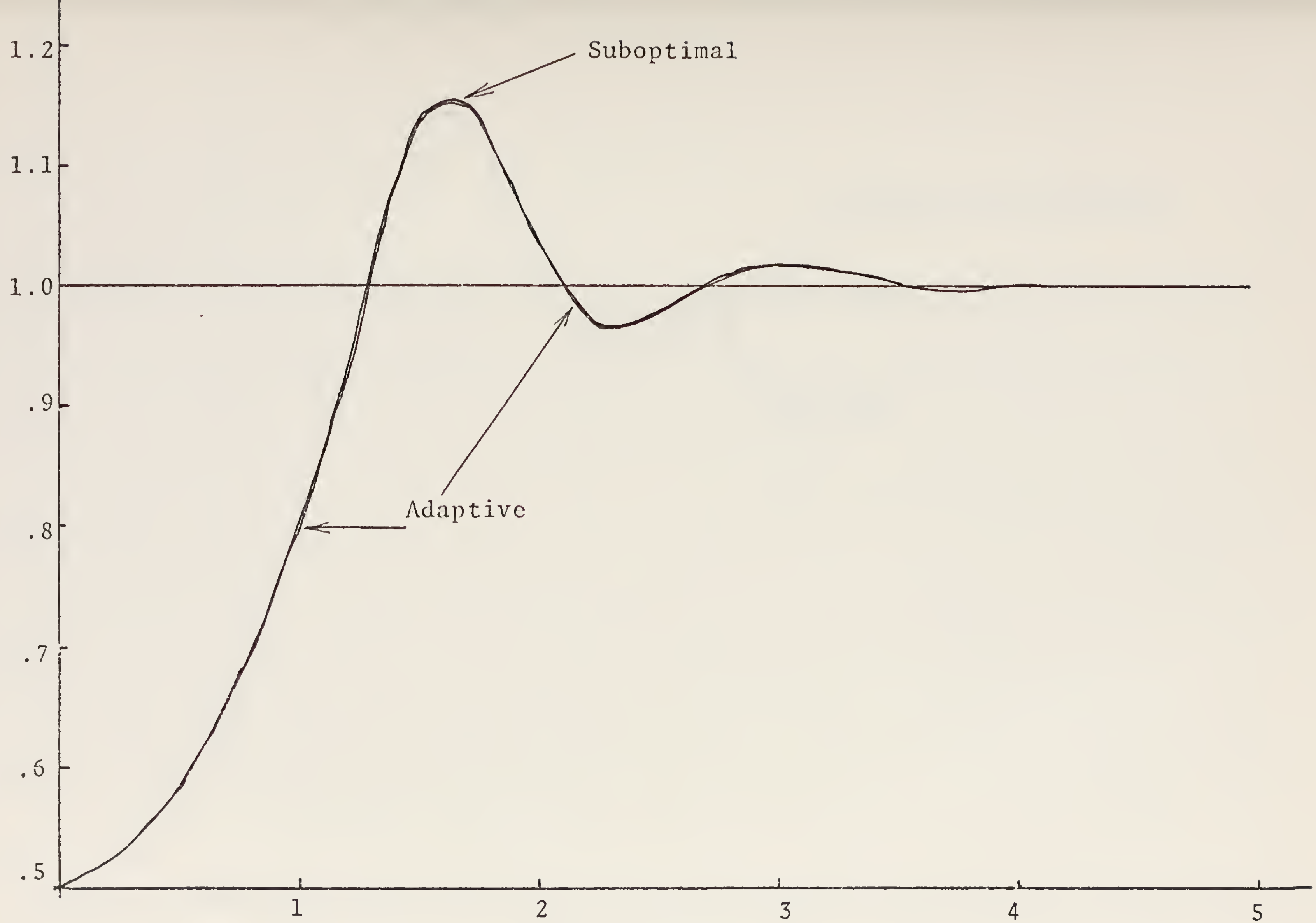


Figure (7.7)
Comparison of System Responses with Adaptive Controller and Non-adaptive Controller

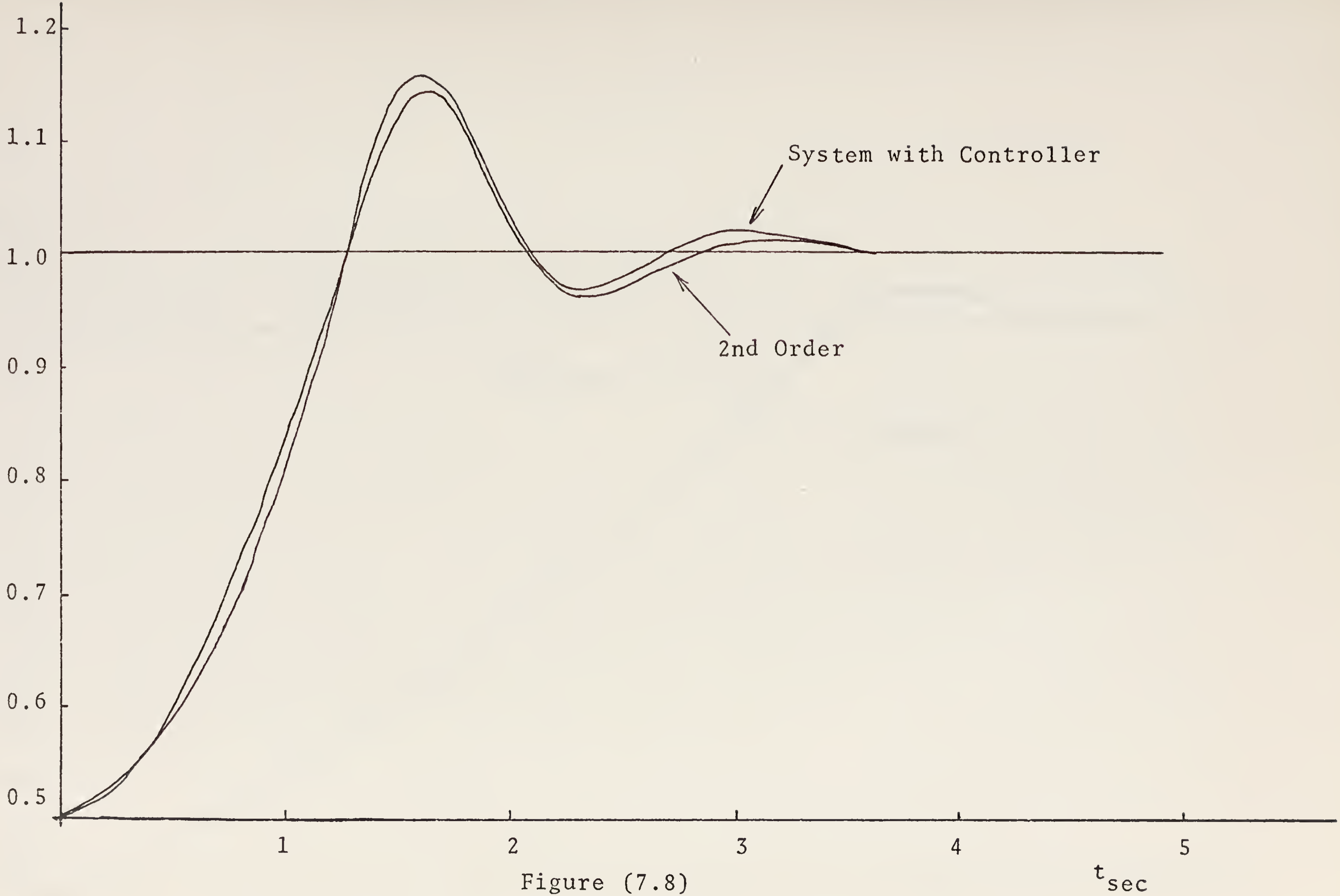


Figure (7.8)

Responses of System and Updated Second-Order Model

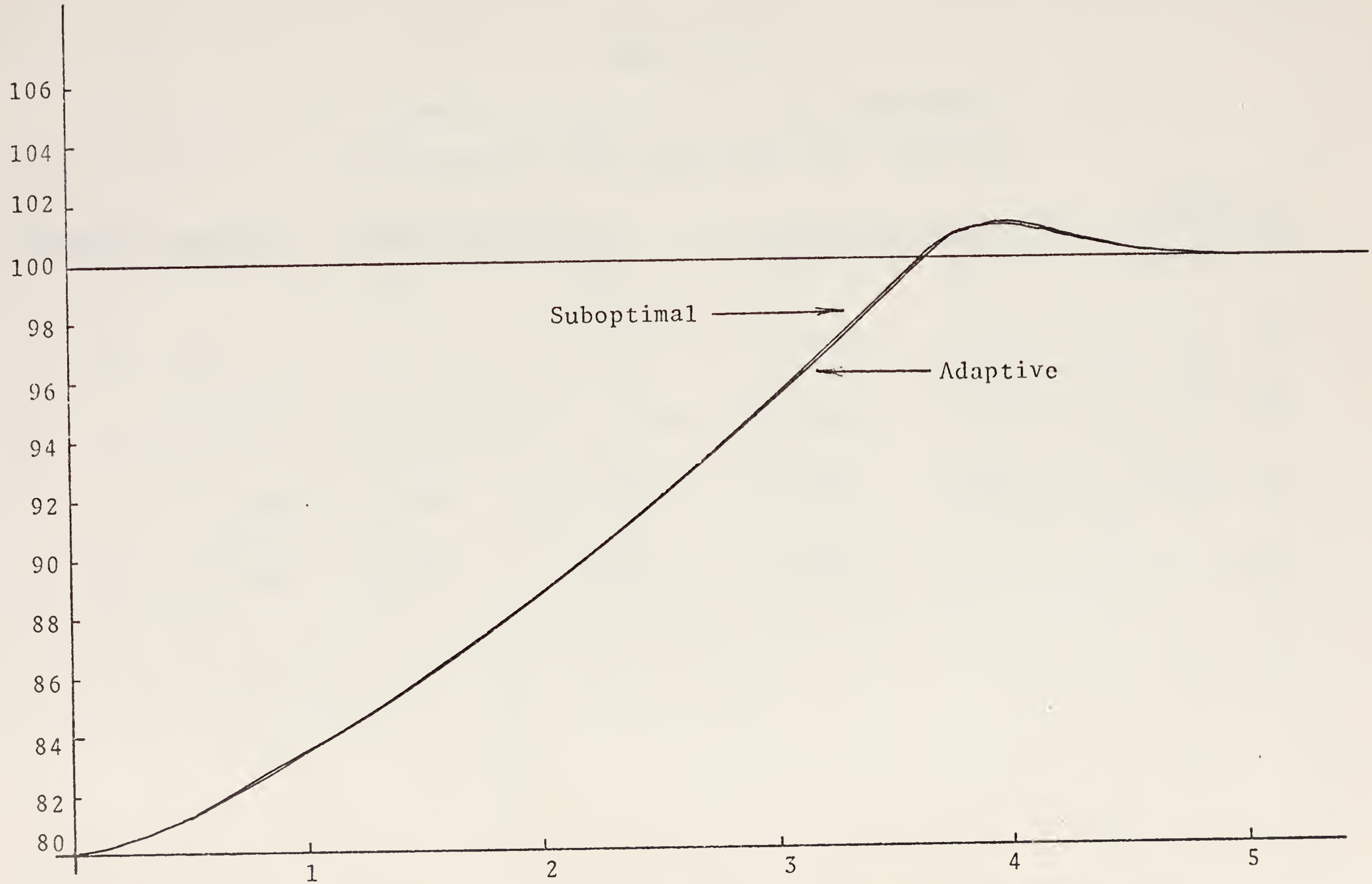


Figure (7.9)
System Responses with Suboptimal Controller
and with the Adaptive Controller

Table (7.2)

Parameter Changes of Model and Controller

during Power Level Transitions (80 - 100% FP)

Adaption Interval	Model Coefficient		Controller Parameter		Minimized Performance Index
	a_0	a_1	k_0	k_1	
1. (0 - 1.5 sec.)	0.65	2.08	0.4142	0.336	7.52×10^{-5}
2. (1.5 - 3.0 sec.)	-2.73×10^{-3}	-0.2156	0.4142	-155.97	2.1×10^{-7}
3. (3.0 - 4.5 sec.)	6.645	2.344	0.4142	0.765	5.88×10^{-4}
4. (4.5 - 6.0 sec.)	-3.075	5.316	0.4142	-0.2	4.9×10^{-5}

VIII. INFLUENCE OF COST FUNCTION

As having been described, the cost function is:

$$J = \frac{1}{2} \int_0^\infty (\underset{\sim}{x}^T \, Q \, \underset{\sim}{x} + r\mu^2) \, dt \tag{8.1}$$

In the previous chapter, the controller parameters were computed for the case that Q is an identity matrix and r equals to unity.

In this chapter, the various weighting factors for the cost functions are considered, depending on the optimal control law. The selection of appropriate values for Q and r is important, and goes to the core of the optimal regulator problem. The process of selection consists of assigning certain values to Q and r, incorporating the resulting controller into the system.

Equations (4.25 and (4.26) of the controller parameters are rewritten for convenience

$$k_0 = -\frac{a_0}{b_0} + \frac{1}{b_0} \left[a_0^2 + \frac{q_1}{r} b_0^2\right]^{\frac{1}{2}}$$

$$k_1 = -\frac{a_1}{b_0} + \frac{1}{b_0} \left[a_1^2 + \frac{q_2}{r} b_0^2 + 2b_0 k_0\right]^{\frac{1}{2}}$$

where q_1 and q_2 are the components of a matrix q, r is the relative weighting factor.

These weighting factors of the cost function decide the controller parameters.

The following values are tried to check the behavior of the actual system.

(1) $\frac{q_1}{r} = \frac{q_2}{r} = 0.1$

The computed controller parameters are:

$$k_0 = 0.0488$$
$$k_1 = 0.1426$$

(2) $\frac{q_1}{r} = \frac{q_2}{r} = 1$

$$k_0 = 0.4142$$
$$k_1 = 0.797$$

(3) $\frac{q_1}{r} = \frac{q_2}{r} = 10$

$$k_0 = 2.317$$
$$k_1 = 3.008$$

(4) $\frac{q_1}{r} = \frac{q_2}{r} = 20$

$$k_0 = 3.583$$
$$k_1 = 4.558$$

These responses are shown in Figure (8.1). The stability of the system with the suboptimal controller depends on weighting factors of the cost function apparently. Since

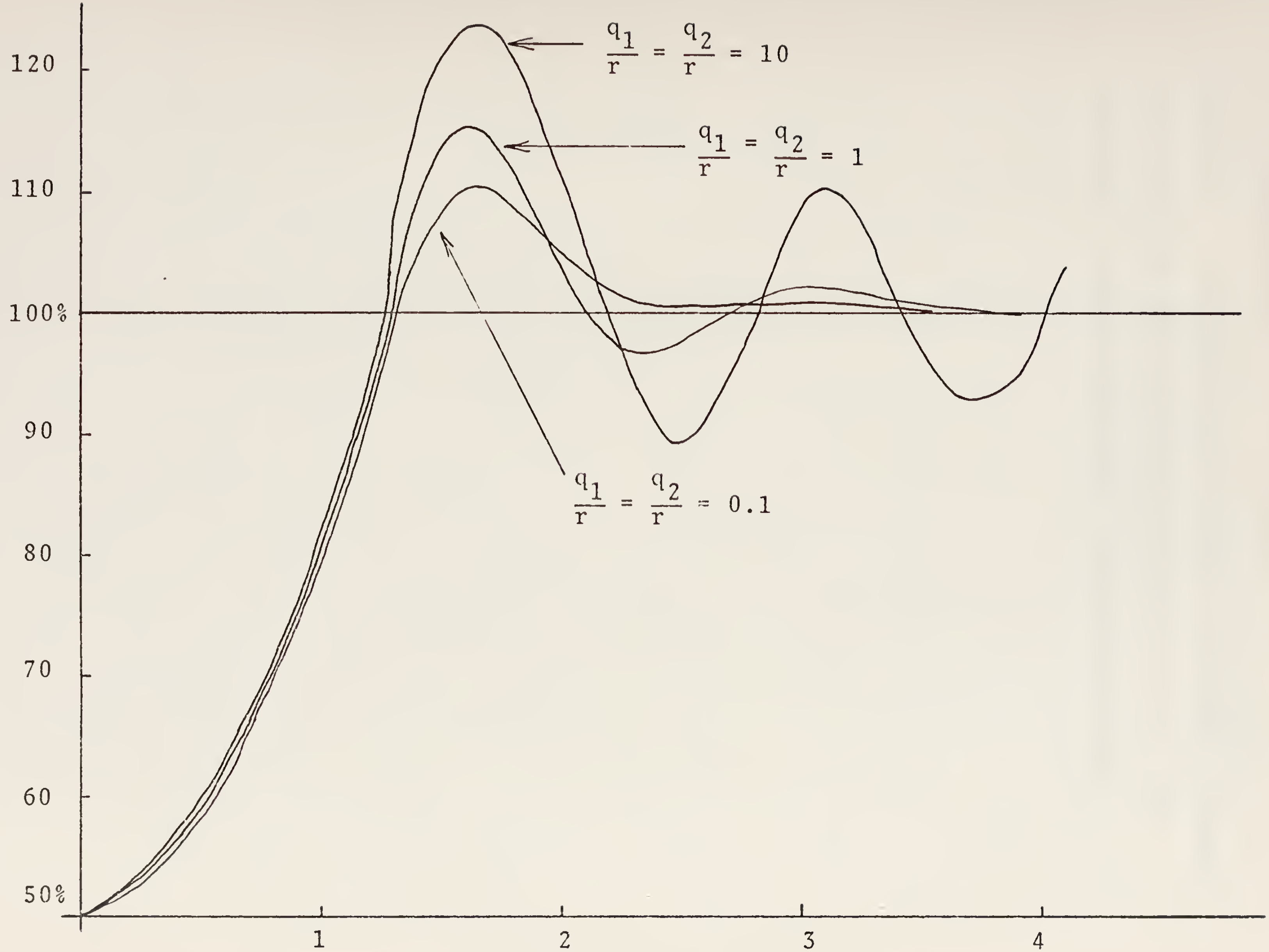


Figure (8.1)
System Response as the Function of the Cost Function

the optimal control law is based on the quadratic cost function, it is essential to select the most reasonable cost function, as described in Reference 14. The resulting fact is consistent with Reference 14.

IX. CONCLUSIONS AND RECOMMENDATIONS

The following conclusions and recommendations are offered as a result of the investigation contained in this report.

A. CONCLUSIONS

1. The simulation of the dynamic behavior of the nuclear reactor system on the digital computer to make feasible this investigation was obtained with some difficulty.

To obtain the stabilized system response required considerable effort, because the system is nonlinear and described by a ninth-order differential equation. The unknown factors such as the specified values of the nuclear reactor parameters complicated the difficulty of implementation of the system equations.

2. In this paper, the gain of the control rod, the reactivity-rate constraint and the effect of the temperature coefficient have been studied. The high gain of the control rod causes the faster response, and the reactivity-rate constraint suitably chosen makes possible the more desirable response regarding the stability, particularly to reduce the deviation from the desired power level. The magnitude of the temperature coefficient has a lesser effect on the system response.

3. It has been shown that the results from the optimal second-order model of the reactor may be used effectively for suboptimal control of the reactor system. This suboptimal controller of the reactor reduces the overshoots of the system response. Considering these overshoots are not desirable practically, its usage provides an improvement in system performance. To find the optimal second-order model, the computer program of search technique is applied to obtain the second-order model parameters.

4. An adaptive control system, which may be realized in practice to control the power level changes of a nuclear reactor, has been proposed.

The parameters of the second-order linear model are continuously updated so that the model accurately represents the behavior of the system. Also, the parameters of the controller are updated to achieve the concept of the adaptive control. The result of this study shows no improvement of the system response using the adaptive controller for the operating condition considered. The adaptive control, therefore, would not have to be on-line, and could be used to update the model parameters to account for slowly changing parameters of the reactor system.

5. The weighting factor of the cost function gives the stability of the system response much influence.

Based on physical considerations, the weighting factor should be chosen as correctly as possible to provide the desired response and this would be a trial and error procedure.

B. RECOMMENDATIONS

Further workers in this area should consider the following facets of nuclear reactor systems.

1. The nuclear reactor system studied in this report did not consider the noise problem. For the purpose of the present paper, it was assumed that an instantaneous noise-free measure of the reactor power level is available. However, as the temperature channel signal indicates the power level of the reactor at an earlier instant, because of the finite transport time between the reactor core and the temperature transducer, this signal has a considerable noise component due to the turbulent coolant flow.

   Therefore, the noise consideration is strongly recommended for future studies.

2. Since the proposed scheme relies on the use of search routines, improvement in the efficiency of the method particularly as applied to digital process computers, would greatly enhance the usefulness of the new technique.

3. In this report, the effectiveness of the adaptive control has not been shown. Consideration should be given to the adaptive controller for different weighting

factors in the cost function to find out whether an on-line adaptive scheme would be required.

4. In much of optimal control, the natural formulation is by state variables. A well-known and popular result is that for the linear regulator problem with quadratic performance index as applied in this study. It has been shown [Ref. 15], however, that the solution is far from realistic or optimum in an engineering sense. Nevertheless, the results and scheme obtained for the linear quadratic-index regulator problem have been forced indiscriminately on the linear time-invariant problem.

Since the weakness of the state-variable formulation in coping with linear feedback control system design is especially apparent in the sensitivity problem, which is one of the primary reasons for the use of feedback, the sensitivity theory by Horowitz and Shaked [Ref. 16] should be investigated.

APPENDIX A

NUMERICAL VALUES OF THE REACTOR MODEL PARAMETERS

$\beta = 6.41 \times 10^{-3}$

$\beta_1 = 2.70 \times 10^{-4}$

$\beta_2 = 7.40 \times 10^{-4}$

$\beta_3 = 2.53 \times 10^{-3}$

$\beta_4 = 1.26 \times 10^{-3}$

$\beta_5 = 1.40 \times 10^{-3}$

$\beta_6 = 2.10 \times 10^{-4}$

$\lambda_1 = 3.014 \text{ sec}^{-1}$

$\lambda_2 = 1.136 \text{ sec}^{-1}$

$\lambda_3 = 0.301 \text{ sec}^{-1}$

$\lambda_4 = 0.111 \text{ sec}^{-1}$

$\lambda_5 = 3.05 \times 10^{-2} \text{ sec}^{-1}$

$\lambda_6 = 1.24 \times 10^{-2} \text{ sec}^{-1}$

$\ell = 10^{-3} \text{ sec}$

$G = 950$

$\tau_m = 0.16 \text{ sec}$

$\tau_t = 12.5 \text{ sec}$

$\alpha_t = -3.5 \times 10^{-3}$

APPENDIX B

DERIVATION OF EQUATIONS ON k_0 AND k_1

The equation is:

$$\underset{\sim}{K}\,\underset{\sim}{A} + \underset{\sim}{A}^T\,\underset{\sim}{K} - \underset{\sim}{K}\,\underset{\sim}{B}\,\underset{\sim}{R}^{-1}\,\underset{\sim}{B}^T\,\underset{\sim}{K} + \underset{\sim}{Q} = 0$$

where

$$\underset{\sim}{A} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \quad \underset{\sim}{B} = \begin{bmatrix} 0 \\ b_0 \end{bmatrix}$$

$$\underset{\sim}{Q} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \quad \underset{\sim}{R} = r \ , \ \underset{\sim}{K} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

with $k_{12} = k_{21}$

Computing term by term

$$\underset{\sim}{K}\,\underset{\sim}{A} = \begin{bmatrix} -a_0k_{12} & k_{11} - a_1k_{12} \\ -a_0k_{22} & k_{21} - a_1k_{22} \end{bmatrix}$$

$$\underset{\sim}{A}^T\,\underset{\sim}{K} = \begin{bmatrix} -a_0k_{21} & -a_0k_{22} \\ k_{11} - a_1k_{21} & k_{12} - a_1k_{22} \end{bmatrix}$$

$$\underset{\sim}{B}\,\underset{\sim}{R}^{-1}\,\underset{\sim}{B}^T = \begin{bmatrix} 0 & 0 \\ 0 & \frac{b^2{}_0}{r} \end{bmatrix}$$

$$\underset{\sim}{K}\ \underset{\sim}{B}\ \underset{\sim}{R}^{-1}\ \underset{\sim}{B}^T\ \underset{\sim}{K} = \frac{b^2_0}{r}\begin{bmatrix} k_{12}k_{21} & k_{12}k_{22} \\ k_{22}k_{21} & k^2_{22} \end{bmatrix}$$

After substituting these matrices into the equation, rearranging, the following four equations are obtained:

$$-a_0\ k_{12} - a_0\ k_{12} - \frac{b^2_0}{r}\ k^2_{12} + q_1 = 0$$

$$-a_0\ k_{22} + k_{11} - a_1\ k_{12} - \frac{b^2_0}{r}\ k_{12}\ k_{22} = 0$$

$$k_{11} - a_1\ k_{21} - a_0\ k_{22} - \frac{b^2_0}{r}\ k_{12}\ k_{22} = 0$$

$$k_{12} - a_1\ k_{22} + k_{21} - a_1\ k_{22} - \frac{b^2_0}{r}\ k^2_{22} + q_2 = 0$$

From the first one of the above four equations

$$\frac{b^2_0}{r}\ k^2_{12} + 2a_0\ k_{12} - q_1 = 0$$

So,

$$k_{12} = \frac{-a_0 + \sqrt{a^2_0 + \frac{b^2_0}{r}\ q_1}}{\frac{b^2_0}{r}} = -\frac{a_0 r}{b^2_0} + \frac{r}{b^2_o}\sqrt{a^2_0 + \frac{q_1}{r}\ b^2_0}$$

Also,

$$\frac{b_0^2}{r} k_{22}^2 + 2a_1 k_{22} - (2k_{12} + q_2) = 0$$

$$k_{22} = \frac{-a_1 + \sqrt{a_1^2 + \frac{b_0^2}{r}(2k_{12} + q_2)}}{\frac{b_0^2}{r}}$$

From the optimal control law,

$$U^* = - R^{-1} \underset{\sim}{B}^T \underset{\sim}{K} \underset{\sim}{X} = -\underset{\sim}{k}^T \underset{\sim}{x}$$

then,

$$k^T = [k_0 \; k_1] = R^{-1} B^T K = \frac{1}{r} [0 \; b_0] \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}$$

$$= \frac{b_0}{r} [k_{21} \quad k_{22}]$$

So,

$$k_0 = \frac{b_0 k_{21}}{r}$$

$$k_1 = \frac{b_0 k_{22}}{r}$$

Therefore

$$k_0 = - \frac{a_0}{b_0} + \frac{1}{b_0} [a_0^2 + \frac{q_1}{r} b_0^2]^{\frac{1}{2}}$$

$$k_1 = - \frac{a_1}{b_0} + \frac{1}{b_0} [a_1^2 + \frac{q_2}{r} b^2{}_0 + 2b_0 k_0]^{\frac{1}{2}}$$

APPENDIX C

OUTLINE OF THE PATTERN SEARCH

As devised by Hooke and Jeeves [Ref. 10], pattern search is a direct search method which results in relatively efficient search along straight ridges or ravines. It is attempted with pattern search to establish the pattern of successful search points in the immediate past from which plausible future search points are predicted. The method for the two-dimensional case is illustrated in Figure (C.1). The search progresses from an initial base point $\underset{\sim}{x}^{\circ}$, and a minimum of $f(\underset{\sim}{x})$ is to be found without the use of derivatives of $f(\underset{\sim}{x})$. A small displacement d_1 from $\underset{\sim}{x}^{\circ}$ in the x_1 direction is effected, and $f(\underset{\sim}{x}_1^{\circ} + d_1, x_2^{\circ})$ is evaluated and compared with $f(x_1^{\circ}, x_2^{\circ})$. If the latter is smaller than the former, a small displacement in the $-x_1$ direction is effected, and $f(x_1^{\circ} - d_1, x_2^{\circ})$ is compared with $f(x_1^{\circ}, x_2^{\circ})$. If the latter is greater than the former, a small displacement d_2 is made from $(x_1^{\circ} - d_1, x_2^{\circ})$ in the x_2 direction, and $f(x_1^{\circ} - d_1, x_2^{\circ})$ is compared with $f(x_1^{\circ} - d_1, x_2^{\circ} + d_2)$. Assuming the latter is the smaller of the two, a pattern move is made in the direction established by the preceding successful moves to the point $\underset{\sim}{x}^1$ in Figure (3.1). At $\underset{\sim}{x}^1$, the small moves which proved to be successful around $\underset{\sim}{x}^{\circ}$ are repeated and if successful lead to a second pattern move to $\underset{\sim}{x}^2$. From $\underset{\sim}{x}^2$, small displacements of magnitude d_1

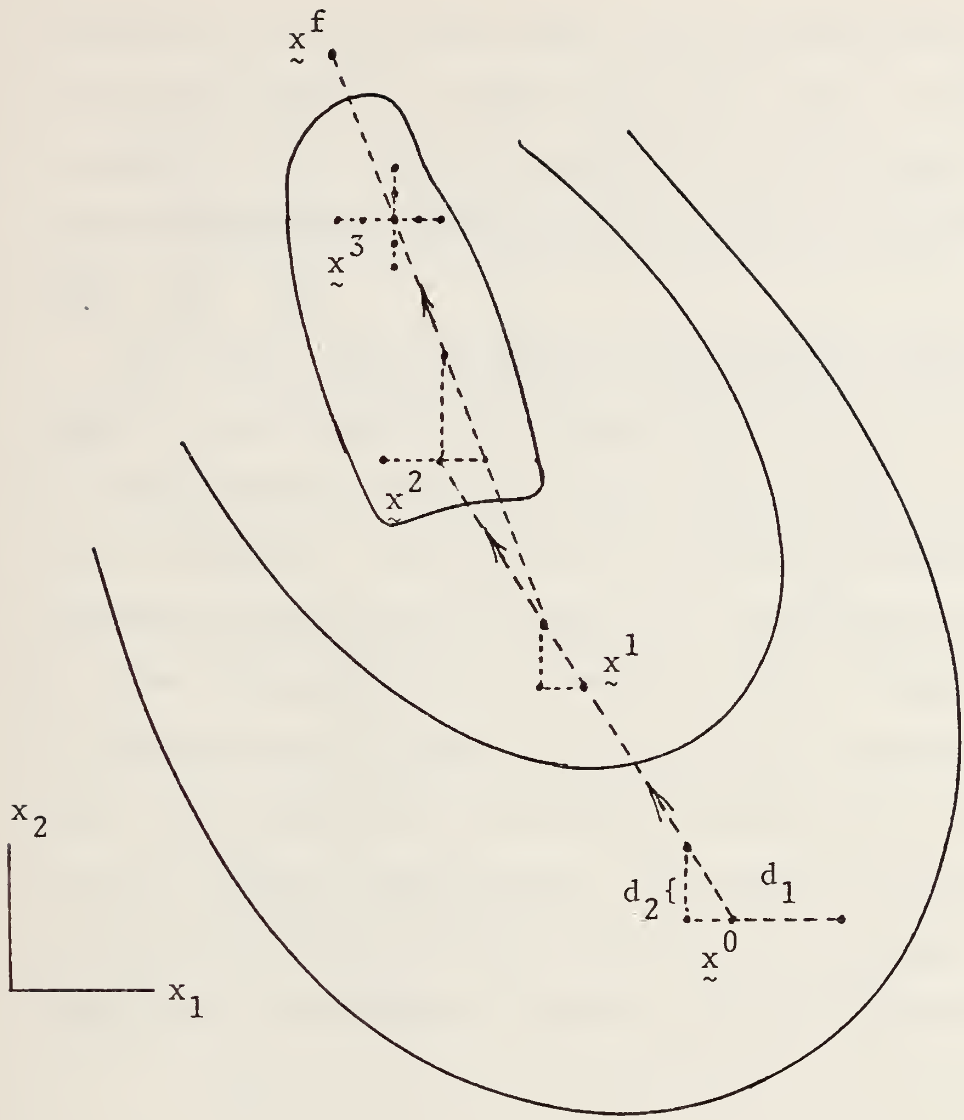


Figure (C1)

Pattern Search

along the x_1 coordinate prove to be unsuccessful, whereas the displacement d_2 yields $f(x_1^2 , x_2^2 + d_2) < f(x_1^2 , x_2^2)$. A pattern move is made therefore along the line which passes through the points $(x_1^1 - d_1, x_2^1 + d_2)$ and $(x_1^2 , x_2^2 + d_2)$.

Note that the step size of the pattern search is made progressively larger as the general directions dictated by the pattern prove successful. When failure occurs, as at point $\underset{\sim}{x}^f$, the step size of the pattern move is reduced as depicted, to point $\underset{\sim}{x}^3$. At $\underset{\sim}{x}^3$, the displacement d_1 from $\underset{\sim}{x}^3$ along the x_1 coordinate fail to improve $f(\underset{\sim}{x})$, as do displacement d_2 from $\underset{\sim}{x}^3$ along the x^2 coordinate. Reductions in d_1 and d_2 are therefore required at this point, and the process starts anew. Ultimately both the displacement and the pattern step sizes are reduced below some preassigned limit, and the search is terminated.

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